The 8th International Congress on Sound and Vibration
2-6 July 2001, Hong Kong, China

- EXISTING REVERBERATION TIME FORMULAE - A COMPARISON WITH COMPUTER SIMULATED REVERBERATION TIMES.

Reinhard O. Neubauer

Ing.-Büro Neubauer VDI
Theresienstr. 28
D-85049 Ingolstadt, Germany
email@Reinhard-Neubauer-VDI.de

Abstract

One of the major objectives of architectural acoustics is to predict the reverberation time in a room. Although, other objective criteria exist to predict acoustical properties of the sound field in a room it is still one of the first investigation, which is made to predict the acoustics of the room. The first approach is the statistical theory based on the assumption of an ideally diffuse sound filed in the room. There are at time several calculation methods in order to compute the reverberation time. First of all, the Sabine, Eyring and Millington-Sette classical equations, as well as later on introduced by Fitzroy, Arau, Tohyama and Nilsson (prEN 12354-6) are investigated in this paper. It is also investigated the Fitzroy-Kuttruff equation and an empirical modified Fitzroy equation. Calculated results are compared with measured reverberation times. Since in recent years, the geometrical acoustics methods of ray tracing and image sources calculations have been very successful implemented using computers. The calculated results using two computer models to simulate reverberation times are compared with calculated results. It is shown that the modified Fitzroy equations yield reliable results in predicting reverberation times in cases where classical reverberation equations predict too short reverberation times; especially in cases of non-uniformly distributed sound absorption.

INTRODUCTION

In solving practical tasks of room acoustics it is a major objective to predict reverberation time adequately correct. Many attempts are made to find an appropriate formula to describe the sound field in rooms with sound absorption distributed on the room surfaces and hence to obtain a reliable basis to derive a correct reverberation time. There is also a need to find more appropriate reverberation time formula due to the ongoing work within the European Standard for rectangular rooms with non-regular distribution of sound absorption as stated in prEN 12354-6. In this paper some of the well-known reverberation time formulae are presented and compared.

CLASSICAL REVERBERATION TIME EQUATIONS

The two most known reverberation time formulae, which are widely used, are Sabine's and Eyring's reverberation time formula followed by the less used Millington-Sette's formula. These well known reverberation time formulae are not depict in detailed, for reference see e.g. [1, 2]

OTHER REVERBERATION TIME EQUATIONS

Among many other reverberation time formulae, the reverberation time formulae under investigation are that of Fitzroy, Arau, Tohyama and that proposed by prEN 12354-6 (i.e. Nilsson model). These formulae are as follows.

Fitzroy's formula

Dariel Fitzroy presented in 1959 a paper [3] introducing his empirically derived equation, which considers non-uniform distribution of sound absorption, which is:

$$T_{60} = \frac{0.16V}{S^2}\left[\frac{-S_x}{ln(1-\bar{\alpha}_x)} + \frac{-S_y}{ln(1-\bar{\alpha}_y)} + \frac{-S_z}{ln(1-\bar{\alpha}_z)}\right] \tag{1}$$

where S_x, S_y, S_z are the total areas of two opposite parallel surfaces in m^2

$\bar{\alpha}_x \bar{\alpha}_y$, $\bar{\alpha}_z$ are the average absorption coefficients of a pair of opposite surfaces

S is the total surface area of the room in m^2.

Arau's formula

Higini Arau presented in 1988 a paper [4] introducing his equation that considers also non-uniform distribution of sound absorption is:

$$T_{60} = \left[\frac{0.16V}{S\bar{a}_x}\right]^{\frac{S_x}{S}}\left[\frac{0.16V}{S\bar{a}_y}\right]^{\frac{S_y}{S}}\left[\frac{0.16V}{S\bar{a}_z}\right]^{\frac{S_z}{S}} \tag{2}$$

where $\bar{a}_x = -ln(1-\bar{\alpha}_x)$, is the average energy absorptivity in x areas, the same for $\bar{a}_y$, $\bar{a}_z$

with $\bar{\alpha}_x = \frac{S_{x1}\alpha_{x1} + S_{x2}\alpha_{x2}}{S_{x1} + S_{x2}}$ is the area-weighted arithmetical mean of the energetic absorption coefficients of the floor S_{x1} and ceiling S_{x2} surfaces, the same is to apply for $\bar{\alpha}_y$ and $\bar{\alpha}_z$

$S = S_x + S_y + S_z$, is the total surface area of the room in m^2.

Tohyama's formula

In [5] Tohyama introduced 1995 an equation, which considers in cases of non-uniform distributed sound absorption in rooms a so-called "Almost-Two-Dimensional" reverberation time formula, which is:

$$T_{Al-xy} = \frac{0.128\ S}{-ln\left(1 - \tilde{\alpha}_{Al-xy}\right)L} \qquad (3)$$

where L is the total surrounding length of the 2-dimensional space.

and $\tilde{\alpha}_{Al-xy} = \bar{\alpha}_{xy}\left(1 - \mu\right) + \bar{\alpha}_z \mu$ is the averaged absorption coefficient in the almost two-dimensional reverberation field

with $\mu = \frac{mc}{L_z^2\, 4f_0}$ is a ration of the number of reflections and $m = \frac{\pi S}{L}$ is the mean free path of the two-dimensional diffuse field, and $\overline{\alpha_z}$ is the averaged absorption coefficient of the z-wall.

The modified Fitzroy equation

R. Neubauer presented during the 8th ISSEM in 1999 [6] a modified Fitzroy equation based on the Eyring correction derived by Kuttruff [7]. This Fitzroy-Kuttruff equation, which considers also non-uniform distribution of sound absorption in rectangular rooms [8], is:

$$T_{60} = \left(\frac{0{,}32\,V}{S^2}\right)\left(\frac{h(l + w)}{\bar{\alpha}^*_{ww}} + \frac{l\ w}{\bar{\alpha}^*_{CF}}\right) \qquad (4)$$

where V, S are Volume in m³ and total surface area of the room in m²
h, l, w are room dimensions height, length and width in m
$\bar{\alpha}^*_{ww}$; $\bar{\alpha}^*_{CF}$ are the average effective absorption exponents of walls, ceiling + floor

with

$$\bar{\alpha}^*_{ww} = -ln\,(1-\bar{\alpha}) + \left[\frac{\rho_{ww}(\rho_{ww}-\bar{\rho})S^2_{ww}}{(\bar{\rho} * S)^2}\right] \qquad (5\text{ a})$$

$$\bar{\alpha}^*_{CF} = -ln\,(1-\bar{\alpha}) + \left[\frac{\rho_{CF}(\rho_{CF}-\bar{\rho})S^2_{CF}}{(\bar{\rho} * S)^2}\right] \qquad (5\text{ b})$$

and $\bar{\alpha}$ is the arithmetic mean of the surface averaged absorption coefficient
$\rho = (1-\alpha)$ is the reflection coefficient

From empirical investigation it was found that the Fitzroy equation may also be modified differently if the room is a cube and the averaged geometrical mean absorption coefficient is $\bar{\alpha}_{geo} \geq 0.2$ or if a flat (i.e. $l/h > 3 \wedge w/h < 3$, ref. [9]) or long room (i.e. $l/h > 3 \wedge w/h < 3 \wedge l/w > 6$, ref. [9]) is investigated.
The modified Fitzroy equation in case of a cube room (i.e. $l = w = h$) and $\bar{\alpha}_{geo} \geq 0.2$ is

$$T_{60} = \left[\frac{0.126 S_x}{-L_x \ln(1-\alpha_x)} + \frac{0.126 S_y}{-L_y \ln(1-\alpha_y)} + \frac{0.126 S_z}{-L_z \ln(1-\alpha_z)} \right]^{1/3} \quad (6\ a)$$

And the modified Fitzroy equation in case of flat and long rooms is

$$T_{60} = \left[\frac{0.126 S_x}{-L_x \ln(1-\alpha_x)} + \frac{0.126 S_y}{-L_y \ln(1-\alpha_y)} + \frac{0.126 S_z}{-L_z \ln(1-\alpha_z)} \right]^{1/2} \quad (6\ b)$$

where

$\bar{\alpha}_x = \frac{S_{x1}\,\alpha_{x1} + S_{x2}\,\alpha_{x2}}{S_{x1} + S_{x2}}$; $L_x = 4(w + h)$; $S_x = 2\,w\,h$; the same applies for the y and z index.

5[th] draft prEN 12354-6, Annex D (Nilsson model)

Recently it was published the 5th draft of the European Standard prEN 12354-6 [10] which offers a method to calculate the reverberation time in cases where the sound absorption in the room is non-uniformly distributed. A short description of the calculation method of this Annex will be given. The aforementioned Annex D indicates possibilities to improve predictions of enclosed spaces, which have non-regular, distributed sound absorption. The model of the Annex D of pr EN 1234-6 is based on the model proposed by Nilsson [11] to deal with aforementioned rooms, i.e. rectangular rooms, by dividing the sound filed in a part that is grazing to the considered surface and a part that is non-grazing, taking due account of the different effect of absorbing materials for those different sound fields. The Annex of prEN 12354-6 provides a practical estimation based on that model of Nilsson but making use of absorption data as measured according to standard methods. For the higher frequencies the total sound field is divided in three grazing fields, grazing to the surfaces perpendicular to each room axis and a diffuse field. For each of these fields the absorption and corresponding reverberation time is determined. For the lower frequencies the total sound field is considered. The absorption area for grazing modes A_x, A_y and A_z, the absorption area for low frequencies A_{xyzd} and the absorption area A_d for the diffuse field are given from Annex D of prEN 12354-6. Here, the equations are shown, neglecting the term for air absorption and the absorption area of objects:

$$A_x = \frac{0.5}{l^2}\left(A_{w1} + A_{w2}\right)\left(\frac{1000}{f}\right)^{1.5} + \left[A_{sidewall,1} + A_{sidewall,2} + A_{Floor} + A_{Ceiling}\right]\sqrt{\frac{2f}{1000}} \quad (7\ a)$$

$$A_y = \frac{0.5}{l^2}\left(A_{sidewall,1} + A_{sidewall,2}\right)\left(\frac{1000}{f}\right)^{1.5} + \left[A_{w1} + A_{w2} + A_{Floor} + A_{Ceiling}\right]\sqrt{\frac{2f}{1000}} \quad (7\ b)$$

$$A_z = \frac{0.5}{l^2}\left(A_{Floor} + A_{Ceiling}\right)\left(\frac{1000}{f}\right)^{1.5} + \left[A_{w1} + A_{w2} + A_{sidewall,1} + A_{sidewall,2}\right]\sqrt{\frac{2f}{1000}} \quad (7\ c)$$

$$A_{xyzd} = \left(A_{Floor} + A_{Ceiling} + A_{w1} + A_{w2} + A_{sidewall,1} + A_{sidewall,2}\right)\sqrt{\frac{2f}{1000}} \quad (7\ d)$$

$$A_d = A_{w1} + A_{w2} + A_{sidewall,1} + A_{sidewall,2} + A_{floor} + A_{ceiling} \quad (7\ e)$$

The reverberation time for each sound field x, y, z and d, as given in Annex D of prEN 12354-6 is, by once again neglecting the term of object fraction and air absorption,

$$T_x = \frac{55.3V}{c_0 A_x};\ T_y = \frac{55.3V}{c_0 A_y};\ T_z = \frac{55.3V}{c_0 A_z};\ T_d = \frac{55.3V}{c_0 A_d} \quad (8\ a)$$

A global estimation of the effective reverberation time is given by the Annex D in [10] as

$$T_{eff} = (T_x + T_y + T_z + T_d) / 4 \quad (8\ b)$$

or for lower frequencies, and neglecting the term of object fraction,

$$T_{eff} = 0{,}16\ V / A_{xyzd} \quad (8\ c)$$

The transition between high and low frequency is given by

$$f_t = \frac{5.8 c_0}{V^{1/3}} \quad (9)$$

If the differences between the four reverberation times from Eq. 8a are small, the diffuse field reverberation time can be considered as an adequate estimation for the considered situation. If not, the effective reverberation time can be taken to be more realistic estimate [10].

COMPARISON OF CALCULATED AND MEASURED RESULTS

A comparison of calculated results using aforementioned formulae and computer simulated results of two different commercial computer programs (CATT-Acoustic [12] and CAESAR [13]) are shown in Figure 1. These results shown are examples of eleven investigated situations with different absorptivities for three different room shapes.

In Figure 1 are depicted results of some critical room shapes with non-uniformly distributed sound absorption. It may be observed that Eq. (6) is very close to the predicted RT using a computer simulation program. Applying classical reverberation time formulae yield too short values. Except of the cube room, where it may be seen that Eqs. (1, 2, 3 and 8) yield also too short RT-values. Close to the computer predicted RT is observed using Eq. (2) (Arau) and the modified Fitzroy equation (Eq. (6)). Especially for the flat and long room situation Eq. (6) turned out to be superior. In the case of a flat room Eq. (3) yield for two investigated situations unreasonable values. This was also observed using Eq. (8) for a cube and a rectangular room situation. In situations where the overall absorptivity was little no large differences were observed using classical RT formulae and the aforementioned equations except of Eq. (8).

In Figure 2 calculated reverberation time is shown for different room volumes from 52 m³ to 1900 m³. There, only mid frequencies of 500 Hz are depicted for comparison. The reverberation time has been measured in-situ in 28 rooms. The rooms were rectangular in shape. In order to compare measured values in "real" rooms with computed values it is necessary to estimate the individual sound absorption coefficients of respective surfaces of the room. This was done by "calibrating" the calculated reverberation time using Sabine's formula and comparing the obtained results with the measured reverberation time yielding the respective sound absorption coefficients. The "calibration" or matching method was done in a kind of engineering survey, i.e. the respective surface, e.g. the floor, ceiling, etc., was given at a first starting point a subjectively judged absorptivity taken from standard absorption coefficients for all six frequency bandwidth from 125 Hz to 4000 Hz. The subjectively judged surface with higher sound absorption coefficients compared to other surfaces, e.g. floor covered with a carpet compared to the plastered walls, of the respective room where than subsequently matched till the calculated reverberation time did correspond to the measured reverberation time. All surfaces of the respective rooms were carefully considered to find the respective sound absorption coefficients. These sound absorption coefficients were then used to calculate the respective reverberation time using the reverberation time formula under consideration. The "real" rooms where empty with some absorbent surfaces. No air absorption is taken into account for computing reverberation time since this air absorption is included due to the calibration procedure in order to get the "real" absorption coefficients. This method was chosen because no standard absorption coefficient was provided for the individual surface material during the in-situ reverberation time measurements. However, Dance and Shield [14, 15, 16, 17] showed that using standard absorption coefficients, i.e. Sabine absorption coefficients, for geometric computer models, yield less accurate prediction results. This was also investigated by Hodgson [18] and it is thought that one cause of the deviation between computed and measured reverberation time may be the applied sound absorption coefficient to room surfaces, particularly where absorption coefficients measured in diffuse sound field conditions and based upon the classical Sabine theory of room acoustics are applied to non-diffuse sound field conditions. From Figure 2 it is observed that the Fitzroy-Kuttruff equation (Eq. (4)) is very close to the measured RT using "calibrated" sound absorption coefficients. The largest difference was observed using Eq. (1) (i.e. Fitzroy) followed by Eq. (3) (i.e. Tohyama). Arau's equation yield for some cases too short and prEN 12354-6 yield at some cases too long RT-values compared to measured RT.

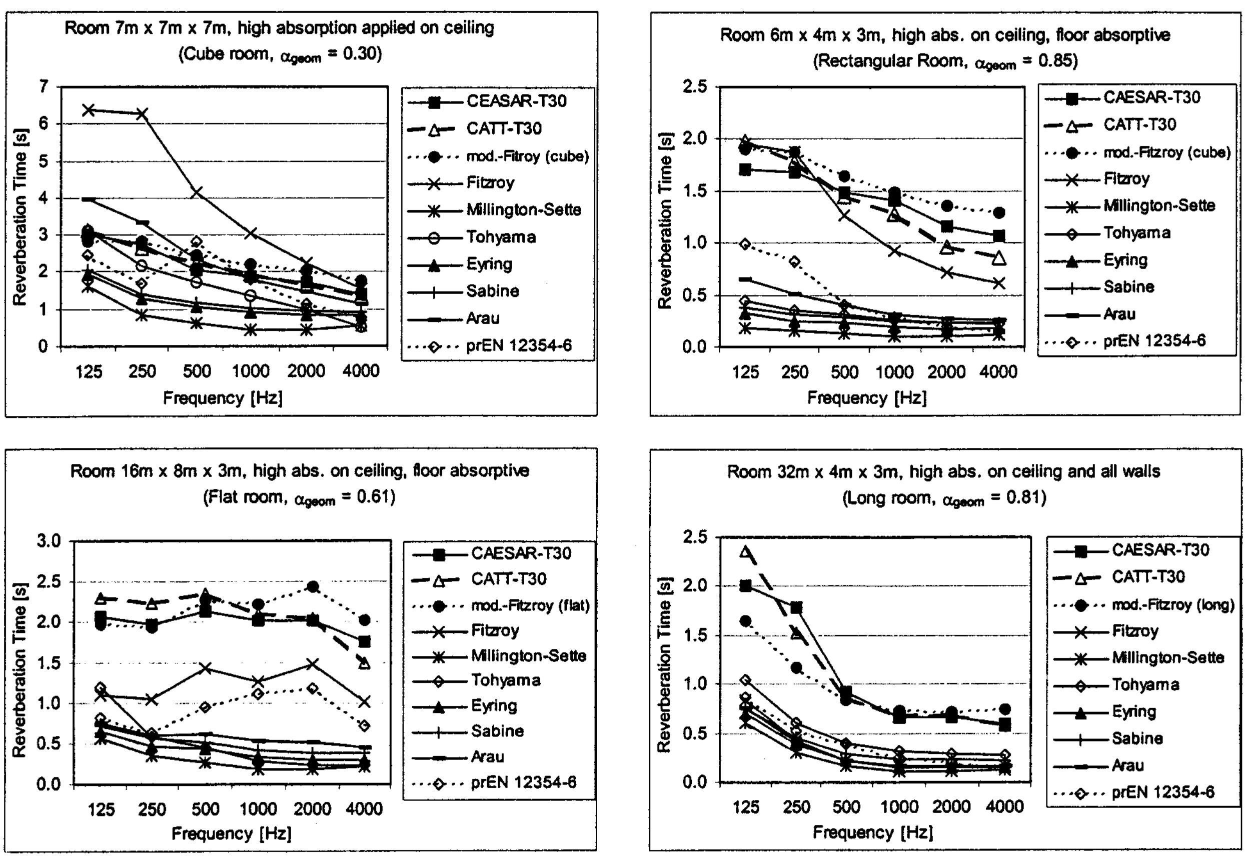

Figure 1. Calculated reverberation times for different room sizes.

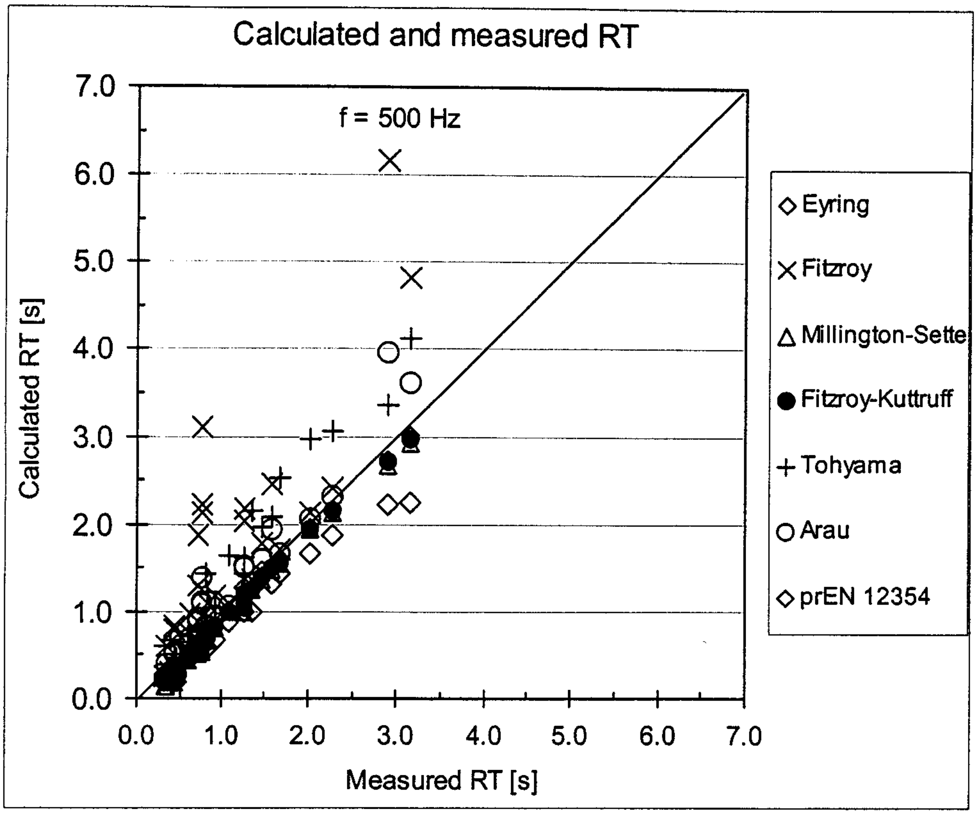

Figure 2. Comparison of calculated and measured RT using calibrated absorption coefficients. Depict are results at 500 Hz for different room volumes.

CONCLUSION

It has been shown that the reverberation time according to a modified Fitzroy equation is useful where the sound absorption at opposite sides are substantially higher than on the rest of the room surfaces. This is e.g. typically for offices where the assumption of diffuse field conditions for applying Sabine's theory are not in agreement with the existing absorber distribution. In this "office-case", i.e. in the case of a "flat-room" Eq. (6 b) could be successfully applied to estimate the reverberation time. This could also be observed in case of a long room, e.g. a hall with absorbing ceiling. Comparison with computer simulated results revealed, compared with the six investigated reverberation time formulae, best results if high absorption is applied. The investigation revealed little differences between computer simulated and predicted results using classical reverberation time formulae if sound absorption in rooms is little. Comparison with measured RT in rectangular rooms using calibrated sound absorption coefficients revealed satisfying agreement. In cases where the overall sound absorption is high using classical reverberation time formulae yield too short reverberation times. In rooms with little absorption, e.g. if only a carpet is considered, the Fitzroy-Kuttruff equation is in agreement with computer simulated results although Eyring's formula works well in that case as well. If the absorption rises, i.e. if the geometrical mean is of about 0.2 or greater, the modified Fitzroy equation as shown in Eq. (6 a) and Eq. (6 b) yield in this investigation best results compared with the classical as well with the investigated reverberation time formulae.

LITERATURE

[1] C.F. Eyring, "Reverberation Time in "Dead" Rooms", *J. Acoust. Soc. Am.*, **1**, pp. 217-241 (1930)

[2] H. Kuttruff,. *Room Acoustics*, 3rd edition, (Elsevier Applied Science 1991)

[3] D. Fitzroy, "Reverberation formulae which seems to be more accurate with non-uniform distribution of absorption", J. Acoust. Soc. Am., **31**, 893-897 (1959)

[4] H. Arau-Puchades, "An Improved Reverberation Formula", Acustica, **65**, 163-180 (1988)

[5] M. Tohyama, H. Suzuki, and Y. Ando, *The Nature and Technology of Acoustic Space*. (Academic Press, London, 1995)

[6] R.O. Neubauer, "Prediction of Reverberation Time in Rectangular Rooms with a Modified Fitzroy Equation", ISSEM'99, 8th International Symposium on Sound Engineering and Mastering, Gdansk, Poland, 115 - 122 (1999)

[7] H. Kuttruff, "Nachhall und effektive Absorption in Räumen mit diffuser Wandreflexion", Acustica, **35**, No 3, 141-153 (1976)

[8] R.O. Neubauer, "Estimation of Reverberation Time in Rectangular Rooms with Non Uniformly Distributed Absorption Using a Modified Fitzroy Equation", 7th ICSV, Garmisch-Partenkirchen, Germany, **3**, pp. 1709 - 1716 (2000)

[9] S. Gruhl, *Beiträge für die Praxis*, Heft 1, (ZIAS, Dresden, 1981)

[10] European Standard prEN 12354-6, February 2001 (CEN/TC126/WG2 N229), 5th draft.

[11] E. Nilsson,. "Decay Processes in Rooms with non-diffuse sound fields", Report TVBA-1004, Lund Institute of Technology (1992)

[12] CATT-Acoustic, room acoustic computer simulation program, www.catt.se

[13] CAESAR, room acoustic computer simulation program, www.akustik.rwth-aachen.de

[14] S. Dance, B. Shield, "Modelling of sound fields in enclosed spaces with absorbent room surfaces." Part I: Performance spaces, *Applied Acoustics*, **58**, pp. 1-18 (1999)

[15] S. Dance, B. Shield, "Modelling of sound fields in enclosed spaces with absorbent room surfaces. Part II: Absorptive panels, *Applied Acoustics*, **61**, pp. 373-384 (2000)

[16] S. Dance, B. Shield, "Modelling of sound fields in enclosed spaces with absorbent room surfaces. Part III: Barriers, *Applied Acoustics*, **61**, pp. 385-397 (2000)

[17] S. Dance, B. Shield, Absorption Coefficients of Common Construction Materials for Use in Computer Modelling of Enclosed Spaces", *J. Building Acoust.*, **7**, pp. 217-224 (2000)

[18] M. Hodgson, "Experimental evaluation of the accuracy of the Sabine and Eyring theories in the case of non-low surface absorption", *J. Acoust. Soc. Am.*, **94**, pp. 835-840 (1993)