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Estimation of Reverberation Time in Rectangular Rooms with Non-Uniformly Distributed Absorption Using a Modified Fitzroy Equation

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ABSTRACT

One of the major objectives of architectural acoustics is to predict the reverberation time in a room. At this time there are several calculation methods to compute the reverberation time, but still Sabine's and Eyring's classical equations are used. In many practical cases the assumption of diffuse sound field conditions for applying Sabine's theory are not in agreement with the existing sound absorption distribution. Therefore, Sabine's formula, as well as other classical reverberation equations like Eyring's or Millington-Sette's, cannot be applied accurately. In 1959 Fitzroy published a paper devoted to the problem of a more accurate calculation of the reverberation time with non-uniformly distributed absorption. In real rooms Fitzroy's equation may therefore become a useful design tool for estimating reverberation time, however, only if it is modified. A modification of Fitzroy's equation is discussed in this paper and some practical examples are presented which compare predicted and measured values of reverberation time in real rooms.

A suggested modification of Fitzroy's equation is presented. Differences between results derived from Fitzroy's, Sabine's, Eyring's and the modified Fitzroy equation as well as results obtained using a room acoustic computer simulation program are compared. Additionally, results are presented and compared using the calculation method of Annex D of prEN 12354-6.

1. INTRODUCTION

In 1959 Dariel Fitzroy published a paper[1] devoted to the problem of more accurate calculation of reverberation time in rooms with non-uniformly distributed absorption. In his paper Fitzroy presented a new formula that afforded results which are closer to those measured in real halls or predicted with the Eyring formula. That is, predicted on the basis of the numbers of times the energy is reflected during a 60-dB decay, reducing the energy by the absorptive coefficient at each reflection, using the mean free path as the average path. Although the results presented were quite significant, it has received no further recognition. For the practical problem of designing a room with a desired reverberation time, an appropriate combination of room volume and interior finishes must be chosen. The reverberation time is predicted in terms of the proposed volume and surface materials of the auditorium. In starting the prediction of reverberation time, it should be noted that the Sabine equation, as well as other classical reverberation equations, is valid only when the sound field in the room can be considered to be sufficiently diffuse.

The most common cause of reduced diffusion is extensive sound absorbing finishes of the ceiling and the floor, and having walls that are all vertical, poorly subdivided, and that reflect sound well. In such rooms, sound waves propagating in directions close to horizontal attenuate slowly, forming a horizontal reverberant field. The reverberation time appears to exceed the value calculated using Sabine's equation.

Since extensive sound absorbing finishes of the ceiling and floor are common in real rooms, Fitzroy's equation may become a useful design tool. A suggested modification of Fitzroy's equation was first presented in a paper by Neubauer[2] and was further discussed during conferences presentations[3, 4]. The differences between results derived from Fitzroy's, Sabine's and the modified Fitzroy equation, as well as those obtained using a room acoustic computer simulation program were compared.

2. FITZROY'S EQUATION

In his paper Fitzroy states that when considering the possibilities suggested by physical acoustics, as contrasted with the geometrical concept, in general, the sound field may tend to settle into a pattern of simultaneous oscillation along a rectangular room's three major axes, the vertical, transverse, and longitudinal. The solution appeared to lie in some relationship between the three possible basic decay rates along these axes, each being influenced by the differing average absorptivities normal to these axes, each rate being unique to its specific axis. In a rectangular room, we are presented with three sets of parallel boundaries. If the energy oscillates simultaneously between each pair of boundaries the average absorption in each pair would control the sound decay. Since the total area of the room's boundaries is exposed to the energy field and influences the decay time to a degree depending upon absorptivity and location, each pair of parallel boundaries can be considered as a specific percentage of the total[1]. The difference between the Eyring formula and the Fitzroy equation is that Eyring's formula assumes the absorptivity to be equal in all directions, and the Fitzroy's equation, in the case of the rectangular room, three different calculations are made using the Eyring formula. But the average absorption is different for each pair of boundaries. Thus, Fitzroy established ratios relating each pair of boundary areas to the total room area. He proposed using only the percentages of the reverberation times for their respective pairs of boundaries. These are added together and the resulting equation yields the reverberation time. Fitzroy's empirically derived equation, which considers non-uniform distribution of absorption, is

$$T_{60}=0.16\,\frac{V}{S^2}\left[\frac{-x}{\ln(1-\alpha_x)}+\frac{-y}{\ln(1-\alpha_y)}+\frac{-z}{\ln(1-\alpha_z)}\right] \tag{1}$$

where x, y, z are the total areas of two opposite parallel walls in m^2

α_x, α_y, α_z are the average absorption coefficients of a pair of opposite walls

S is the total surface area of the room in m^2

3. THE EYRING-KUTTRUFF EQUATION

C.F. Eyring pointed out in his paper[5] that Sabine's formula is essentially a "live" room formula and that the reverberation time varies somewhat with the shape of the room. Eyring presented a thoroughly revised theory and derived a form of reverberation time equation, which is more general than Sabine's formula. He considered the difference between the basic assumptions leading to Sabine's and to his formula as well as some experimental data that supports his more general type of equation. In his paper Eyring also indicated that no one formula, i.e. the Sabine or the Eyring formula, are essentially all-inclusive. They need some modification. Eyring's formula, which is extensively discussed in the literature, is

$$T_{60} = \frac{0.16V}{-S\,ln(1-\overline{\alpha})} \tag{2}$$

with

$$\overline{\alpha} = \frac{1}{S}\sum_i S_i\alpha_i \tag{3}$$

Eyring's equation has been derived on the assumption that the sound field in the enclosure is considered to be ideally diffuse. Usually, it yields sufficiently accurate results, although it is based upon the assumption, that equally sized wall areas are hit by the same quantity of energy per second. This is the case if the sound field in the room is diffuse or homogeneous, meaning that in each point the directional distribution of energy transport is uniform. Since this condition would result in vanishing net energy flow, perfect sound field diffusion is incompatible with any wall absorption. Strictly speaking, there must be at least some energy flow from the sound source towards the wall. On the other hand, a non-uniform distribution of wall absorption may impair sound field homogeneity so severely that no accurate results from application of Eyring's formula, can be expected. Dance and Shield[6] have shown that the Eyring formula produced inaccurate predictions when absorbent material was unevenly distributed over room surfaces[7].

Kuttruff[8] presented a paper in which a correction to Eyring's formula is derived, assuming of a diffuse sound reflection from each wall element. The concept of reflection coefficient is used:

$$\rho = 1-\alpha \tag{4}$$

On the assumption that the absorption coefficient α and hence ρ are independent of angle Kuttruff made use of Lambert's law of diffuse reflection. By focussing on the overall reverberation time, neglecting details of the decay process and assuming an exponential law for the time dependence of the irradiation strength, Kuttruff defined an absorption exponent α^*, and showed that the absorption exponent would assume its Eyring value if the irradiation strength were constant[8].

$$\alpha_{Eyring} = -ln\,\overline{\rho} = -ln\,(1-\overline{\alpha}) \tag{5}$$

This is true if ρ and hence the absorption coefficient α has the same value everywhere[9]. In general however, the effective absorption exponent will be smaller or larger than $(-ln\,\overline{\rho})$, depending on the room shape and on the distribution of the wall absorption. Kuttruff's assumption using Lambert's law of diffuse reflections, and Gilbert's iteration scheme, lead to the correction formula mentioned[8, 9]. Kuttruff's absorption exponent is

$$\alpha^* = ln\left(\frac{1}{\overline{\rho}}\right) + ln\left(1 + \frac{\sum\limits_{n} \rho_n\,(\rho_n - \overline{\rho})\,S_n^2}{(\overline{\rho}S)^2 - \sum \rho_n^2\,S_n^2}\right) \tag{6}$$

where $\overline{\rho} = 1-\overline{\alpha}_n$ denotes the average reflection coefficient of surface area S_n
S is the total surface area of the room in m^2.

In most cases the second term in the denominator is much smaller than the first and hence can be neglected[9]. Expanding the second logarithm into a power series and neglect all terms of higher than first order yields

$$\alpha^* \approx \alpha_{Eyring} + \frac{\sum\limits_{n} \rho_n\,(\rho_n - \overline{\rho})S_n^2}{(\overline{\rho}\,S)^2} \tag{7}$$

Inserting Kuttruff's correction into Eyring's reverberation formula and completing this formula by taking into account the attenuation constant m of air, yields the Eyring-Kuttruff equation

$$T_{60} = \frac{0.16V}{S[-ln(1-\overline{\alpha})] + \Delta + 4mV} \tag{8}$$

whereΔ is

$$\Delta = \frac{\sum\limits_{i} \rho_i(\rho_i - \overline{\rho})S_i^2}{(\overline{\rho}\,S)^2 - \sum\limits_{i} \rho_i^2\,S_i^2} \tag{9}$$

Kuttruff showed that his correction to the Eyring formula could easily be applied to the case where i-1 surfaces have nearly the same reflection coefficient and one surface, namely the ith surface, e.g. the floor on which the audience sits, has a different reflection coefficient[9]. The results he presented showed good agreement with computer simulated results.

4. A CORRECTION TO FITZROY'S FORMULA

Since Kuttruff [8, 9] introduced a correction to the Eyring formula and Fitzroy's equation uses the Eyring concept, it is proposed to apply with the same correction to Fitzroy's equation[2]. On the assumption that in real rooms the main absorption is on the floor or on the ceiling or on both, the Fitzroy equation may then be modified into an equation more convenient for practical use. Splitting the Kuttruff correction into two parts, namely the part for the ceiling-floor and the part for the walls, and introducing the modified Kuttruff correction to the Fitzroy equation, yields to the modified Fitzroy equation.

5. THE MODIFIED FITZROY EQUATION

Because the examples Fitzroy gives in his paper work well with the three-term formula, it is worthwhile to attempt to use it to predict the overall reverberation time, for comparison with classical reverberation time formulae. Comparison of calculation results reveal, however, no advantage over the simpler equations provided by Sabine or Eyring[4]. In particular, calculations using different room volumes and room absorptivities, i.e. others than Fitzroy provides in his paper, reveal results the other way around. Using the Fitzroy equation provides much higher values of reverberation time than using Sabine or Eyring. For the important practical case where either the ceiling and/or the floor is highly absorptive we have an "almost two-dimensional" field. This occurs in general in real rooms when the absorbing capacity of the ceiling and floor exceeds those of the walls, Fitzroy's equation may then be written in a modified manner. If we divide the floor and ceiling area and the remaining wall areas one get expressions as follows.

Surface of the ceiling and the floor: $\mathrm{S}_{\mathrm{CF}} = 2lw$ (10 a)

Surface of the walls: $S_{WW} = 2lh + 2hw$ (10 b)

Total surface area of the room: $S_{\mathrm{total}} = 2[h(l + w) + lw)]$ (10 c)

Using Equation 10a 10b and 10c above, we may rewrite the Fitzroy equation, and introducing Kuttruff's modified correction of Equation 7, yields the new formula (NF)

$$\mathrm{T}_{60} = \left(\frac{0.32V}{S^2}\right) \left(\frac{h(l+w)}{\overline{\alpha}^*_{ww}} + \frac{lw}{\overline{\alpha}^*_{CF}}\right) \tag{11}$$

where V, S are the room volume in m^3 and the total surface area of the room in m^2, respectively.

h, l, w are the room dimensions; height, length and width in m, respectively.

$\overline{\alpha}^*_{ww}$; $\overline{\alpha}^*_{CF}$ are the average effective absorption exponent of the walls, and the ceiling / floor, respectively.

with

$$\overline{\alpha}^*_{ww} = -ln(1-\overline{\alpha}) + \left[\frac{\rho_{ww}(\rho_{ww}-\overline{\rho})S^2_{ww}}{(\overline{\rho}S)^2}\right] \tag{12 a}$$

$$\overline{\alpha}^*_{CF} = -ln(1-\overline{\alpha}) + \left[\frac{\rho_{CF}(\rho_{CF}-\bar{\rho})S^2_{CF}}{(\bar{\rho}S)^2}\right] \qquad (12\ b)$$

and $\overline{\alpha}$ is the arithmetic mean of the surface averaged absorption coefficient
$\rho = (1-\alpha)$ is the reflection coefficient

A comparison of results calculated using the new formula and Eyring's formula are shown in Figure 1, for mid-frequencies of 500 Hz, where the sound absorption of air is neglected. The reverberation times in Figure 1 are for different room volumes, but with an overall surface absorptivity of 0.02 (live room conditions). Eyring's reverberation time is about 1% less than the reverberation time using the new formula (Fitzroy-Kuttruff equation. Sabine and the new formula give the same values of reverberation time if the overall absorption of the room is low.

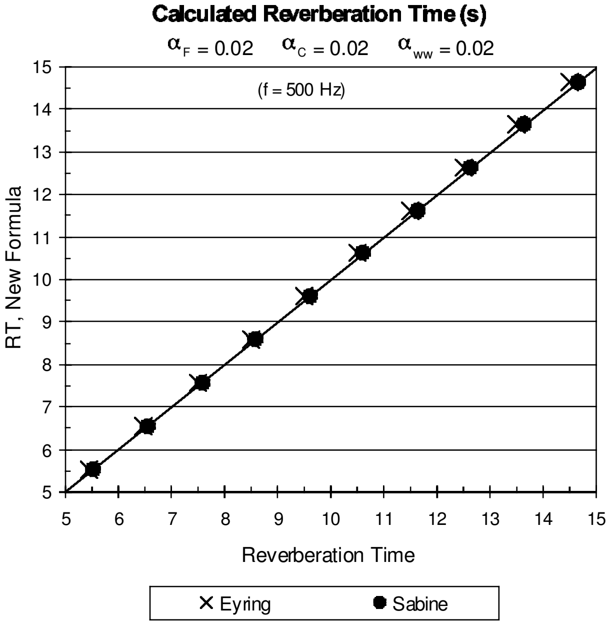

Figure 1. Calculated reverberation time for low absorption on all room surfaces. The line shows values predicted using the new formula, plotted against itself.

A comparison of results for "dead room conditions", i.e. an overall surface absorptivity of 0.95, are shown in Figure 2. Eyring's reverberation time is close to the new formula, whereas Sabine yields reverberation times about 68 % higher than those using Eyring's formula.

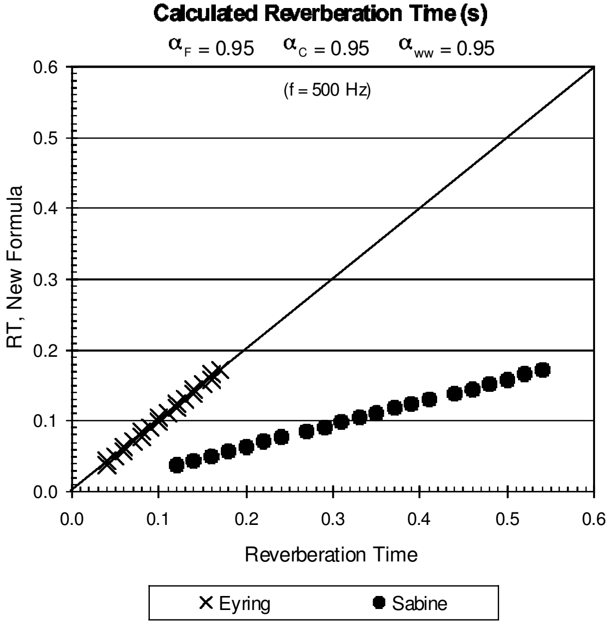

Figure 2. Calculated reverberation time for high absorption of all room surfaces. The line shows values predicted using the new formula, plotted against itself.

Sabine's formula is essentially a "live-room" formula and as shown in the literature[5, 10] the Sabine reverberation time varies somewhat with the shape of the room. Sabine's formula assumes perfectly diffuse conditions of the energy density, i.e. a uniform distribution and random flow of energy. The more general formula for calculating reverberation time, that of Eyring (Equation 2) is based on the mean free path between reflections. Since both Sabine's formula and Eyring's assume diffuse sound field condition one may replace the absorptive properties of the individual surfaces with a room average, so that they can be equated to give

$$\overline{\alpha} = -ln(1-\overline{\alpha}) \qquad (13)$$

The relative difference between the quantities is shown graphically in Figure 3.

From the graph in Figure 3 it is apparent that if $\overline{\alpha} << 1$, both formulae give similar values of calculated reverberation time (allow an error of about 10 %). However, for $\overline{\alpha}$ increasing to 0.3, 0.4, 0.5, etc. $-\ln(1-\overline{\alpha})$ becomes 0.36, 0.51, 0.67, etc., i.e. the difference becomes appreciable when $\overline{\alpha}$ is greater than 0.2. Since the Eyring RT is less than the Sabine RT, it implies a more rapid decay of sound in the room. For small values of absorption in a room, however, it is immaterial whether we use Sabine or Eyring to calculate reverberation time. The Eyring formula reduces to the Sabine formula for $\overline{\alpha} << 1$.[5, 11]

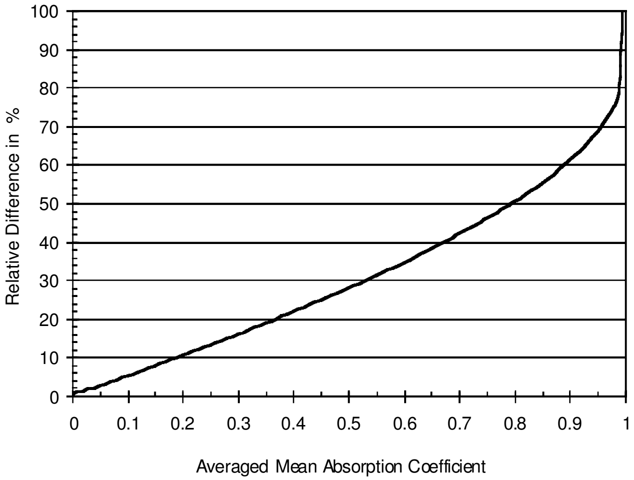

Figure 3. Calculated percentage difference between $-\ln(1-\overline{\alpha})$ and $\overline{\alpha}$.

By looking the equation of Sabine it is clear that if the average absorption coefficient increases Sabine's formula will, in the limit, lead to an unreasonable value. This is, if the absorption in the room is overall near unity, in a "dead" room, Sabine's formula, contrary to Eyring's formula, gives a finite reverberation time which is nonzero but simply reduces to a constant proportional to

$$T \sim V/S \tag{14}$$

which is a function of the room shape.

The results using the modified Fitzroy equation (the new formula and those using the Sabine and Eyring equations, for non-uniformly distributed sound absorption in the room is shown in Figure 4. Here, most absorption is on the floor, some on the ceiling and only minor absorption at the walls.

It is informative in some respects to compare measured reverberation time values (T-30) and computed results using the Fitzroy equation and the new formula (the Fitzroy-Kuttruff equation).

All measurements are performed according to the requirements of German Standard DIN 52216[12], DIN EN ISO 3382[13]. Measurements were taken at at least 3 locations with at least two microphone positions. The experimental arrangement consisted of a $^1/_2$" microphone and a noise source mounted inside the room. The subsequent analysis was achieved conveniently using a dual channel real-time analyser (Norsonic RTA Type 840-2). The reverberation time was calculated using a linear least-squares regression of the measured decay curve, from a level 5 dB below the initial level, to 35 dB below, using an automatic evaluation and linear regression of ensemble-averaged decay curves.

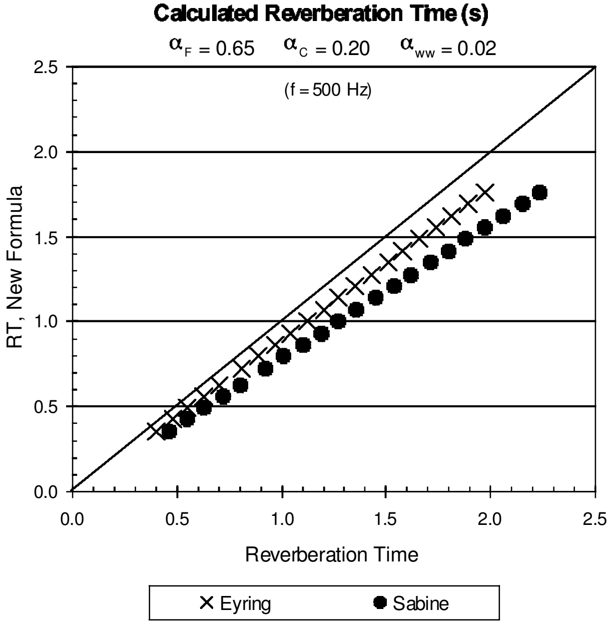

Figure 4. Calculated reverberation time for non-uniformly distributed sound absorption. The line shows values predicted using the new formula, plotted against itself.

The reverberation time was measured in 26 rectangular rooms. Details of the room dimensions and measured reverberation times for the 500 Hz and 1 kHz octave bands are shown in Table 1.

A comparison of the measured reverberation time values (T-30) and the computed results using the Fitzroy equation and the new formula (the Fitzroy-Kuttruff equation) is shown in Figure 5. To compare measured values in "real" rooms with computed values it is necessary to estimate the sound absorption coefficients of individual surfaces of the room. This was done by "calibrating" the calculated reverberation time using Sabine's formula and comparing the obtained results with the measured reverberation time to yield the sound absorption coefficients. The "calibration" or matching method was done in a form of engineering survey, i.e. the respective surfaces of the floor, ceiling, etc. were given, as a first starting point, a subjectively estimated absorptivity taken from standard absorption coefficients data, for all six frequency bands from 125 Hz to 4000 Hz. The surface adjudged to have the highest sound absorption coefficients compared to other surfaces, e.g. a carpet covered floor compared to the plastered walls, was subsequently matched until the calculated reverberation time corresponded to the measured reverberation time. All surfaces of the rooms were then carefully considered to find their respective sound absorption coefficients. These sound absorption coefficients were then used to calculate the reverberation time using the reverberation time formula under consideration. The “real” rooms were empty with some absorbent surfaces. No air absorption was taken into account for computing reverberation time since air absorption is included in the matching procedure, in order to get “real” absorption coefficients.

Table 1. Details of the rooms investigated, and measured reverberation times for the frequency 500 Hz and 1 kHz octave bands.

No	Volume	Length	Width	Height	Measured RT [s]	
	V [m^3]	l [m]	w [m]	h [m]	f = 500 Hz	f = 1 kHz
1	52.13	4.45	3.30	3.55	1,27	1,16
2	62.17	5.19	3.72	3.22	1,47	1,09
3	68.03	5.80	4.60	2.55	2,03	1,92
4	73.03	7.27	3.44	2.92	0,34	0,37
5	94.24	6.30	5.40	2.77	0,80	0,72
6	114.96	6.07	5.90	3.21	1,60	1,23
7	131.06	6.23	5.45	3.86	0,44	0,45
8	138.90	6.62	5.45	3.85	0,43	0,39
9	149.13	7.12	6.89	3.04	1,67	1,21
10	156.92	7.26	6.53	3.31	0,75	0,77
11	161.08	8.55	6.28	3.00	0,71	0,87
12	194.04	9.24	7.00	3.00	0,61	0,56
13	196.91	8.87	7.40	3.00	1,09	0,93
14	203.00	8.49	7.97	3.00	0,75	0,84
15	220.11	7.40	6.61	4.50	2,27	2,70
16	241.87	9.31	8.66	3.00	0,77	0,85
17	270.00	11.25	8.00	3.00	1,35	1,34
18	323.97	13.83	8.05	2.91	0,38	0,36
19	323.97	13.83	8.05	2.91	0,32	0,27
20	334.96	13.42	8.32	3.00	0,72	1,18
21	379.01	16.00	7.52	3.15	0,62	0,56
22	750.00	15.00	10.00	5.00	1,26	0,81
23	750.00	15.00	10.00	5.00	0,84	1,23
24	1,100.55	14.50	11.50	6.60	0,93	1,53
25	1,899.68	24.62	12.00	6.43	3,15	2,79
26	1,899.68	24.62	12.00	6.43	2,89	2,39

This method was chosen because no absorption coefficient date was available for the individual surface materials for the rooms where the reverberation time was measured. However, Dance and Shield[6, 7, 14, 15] showed that using standard absorption coefficients, i.e. Sabine absorption coefficients, for geometric computer models, gives less accurate prediction results. This was also investigated by Hodgson[16], who concluded that one cause of the deviation between computed and measured reverberation time may be that the sound absorption coefficient data ascribed to the room surfaces, particularly where the absorption coefficients were measured in diffuse sound field conditions and based upon classical room acoustics Sabine theory, were applied to non-diffuse sound field conditions. The accuracy, indicated by the agreement between measured and predicted reverberation times, with which the absorption coefficients could be estimated using the "calibration" procedure is shown in Table 2.

Table 2. The average error of the estimated reverberation time using the "calibrated" absorption coefficients. Shown are the mean error, and the standard deviation of the mean.

Average error in reverberation time, in octave bands					
125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
0.14 ± 0.02	0.13 ± 0.02	0.20 ± 0.04	0.19 ± 0.03	0.16 ± 0.02	0.09 ± 0.02

From Table 2 it is apparent that predicted reverberation times could not be estimated closer than approximately 0.07 seconds, compared to the measured reverberation times. This is because the estimated sound absorption coefficient is quoted to two decimals places. For some configurations in this investigation, zero difference in reverberation times could be obtained but some could not estimated closer than about 0.6 % using the "calibrating" procedure.

Having obtained the sound absorption coefficients using the "calibrating" procedure it was then proposed to use these to calculate reverberation times in the octave frequency bands of interest. The calculation was carried out using the Fitzroy and the modified Fitzroy equation (Fitzroy-Kuttruff-Equation). A comparison of the calculation results for the investigated frequency bands are presented in Figure 5.

Figure 5 shows a comparison of reverberation times calculated using the Fitzroy equation and the modified Fitzroy equation (Fitzroy-Kuttruff-Equation, "NF"), using matched sound absorption coefficients. The improvement to the Fitzroy RT in the different frequency bands, achieved by applying the modified Kuttruff correction is apparent. The Fitzroy-Kuttruff equation predicts values that are closer to the measured values. The percentage difference between Fitzroy and Fitzroy-Kuttruff reverberation times are shown in Figure 6.

The percentage difference between the Fitzroy and the New Formula reverberation times shown in Figure 6 are calculated using:

$$\frac{RT(Fitzroy) - RT(NF)}{RT(Fitzroy)}\,100\% \qquad (15)$$

where RT(NF) is the calculated reverberation time using the new formula
RT(Fitzroy) is the calculated reverberation time using the Fitzroy equation

From Figure 6 it is observed that values obtained using the new formula (Fitzroy-Kuttruff) are generally lower than those using the Fitzroy equation. This result is well known, i.e. it is frequently the case that calculated reverberation times using the Fitzroy equation are too long, compared to measured results. Figure 7 shows the relative difference using Fitzroy and the new formula compared to measured reverberation times.

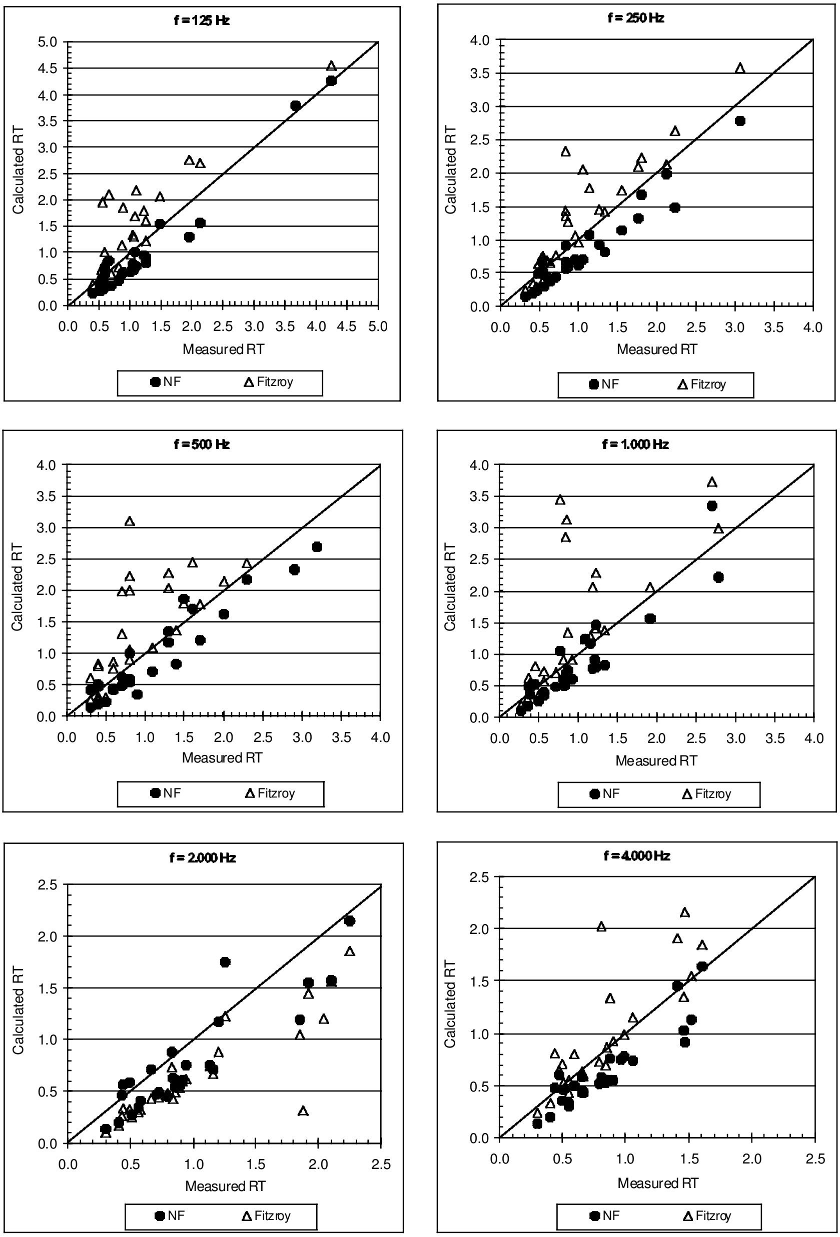

Figure 5. Calculated and measured reverberation times (RT) using matched sound absorption coefficients. Shown are results using Fitzroy and the Fitzroy-Kuttruff equation (NF). The line shows measured reverberation time, plotted against itself.

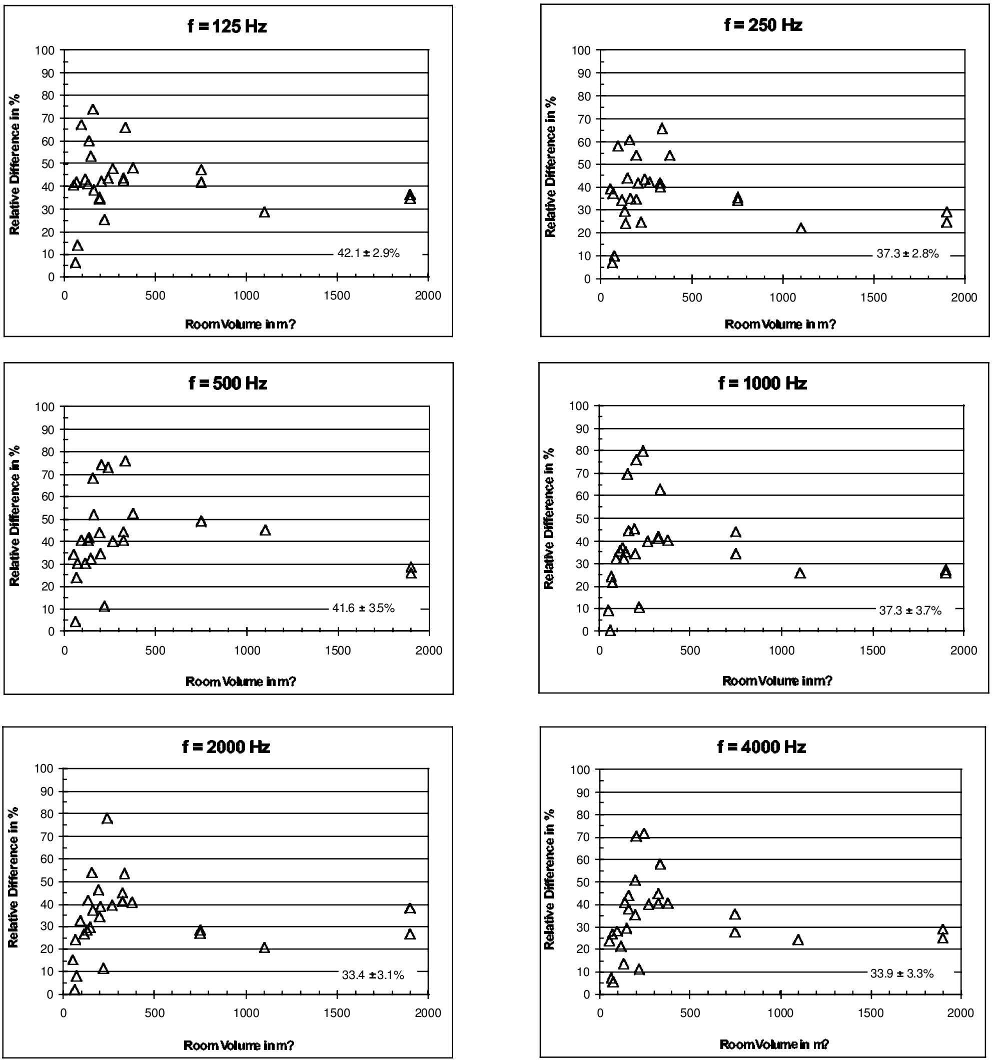

Figure 6. Relative difference of the reverberation times calculated using the Fitzroy and Fitzroy-Kuttruff (NF) equations.

Upgrading the Fitzroy equation using the modified Kuttruff correction shows, on average, an improvement of approximately 38 %. In contrast to the Fitzroy equation, no unreasonable value of predicted reverberation times occurred using the modified Fitzroy-Kuttruff equation. This is clearly observed from Figure 5.

Since computer simulated room acoustic calculations are widely used successfully, it is of some interest to compare simple calculated results and computer simulated results. In make a comparison the same "calibrated" sound absorption coefficients were used both for the new formula calculation, and for the computer simulation procedure. The room acoustic computer simulation program used was CATT-Acoustic v7.1[17]. This software is a room acoustic prediction program based on the Image Source Model, Ray-tracing and Randomised Tail-corrected Cone-tracing. This is considered to be one of the most reliable computer programs[18, 19]. Figure 8 shows a comparison of measured reverberation times and those calculated using the CATT-Acoustic program and the Fitzroy-Kuttruff equation, using the same matched or "calibrated" sound absorption coefficients. The results in Figure 8 show good agreement for all six octave-band frequencies from 125 Hz up to 4 kHz. In general, however, reverberation times predicted by the computer program, as well as by the Fitzroy-Kuttruff equation, are shorter than the measured reverberation times. The rectangu-

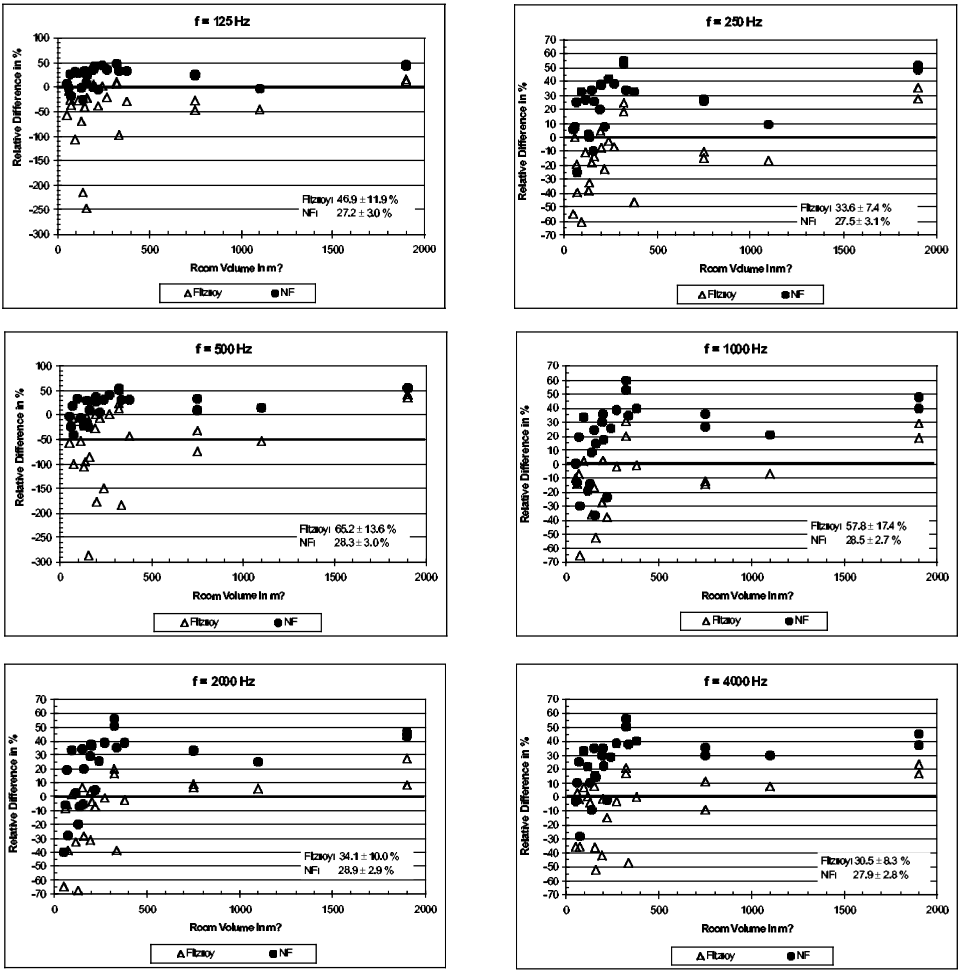

Figure 7. Relative difference of the calculated RT using Fitzroy and Fitzroy-Kuttruff (NF) compared to the measured reverberation time.

lar room volumes shown in Table 1 ranged from about 52 m^3 up to about 1900 m^3. Predicted reverberation times, obtained by the CATT-Acoustic program was the T-30 value.

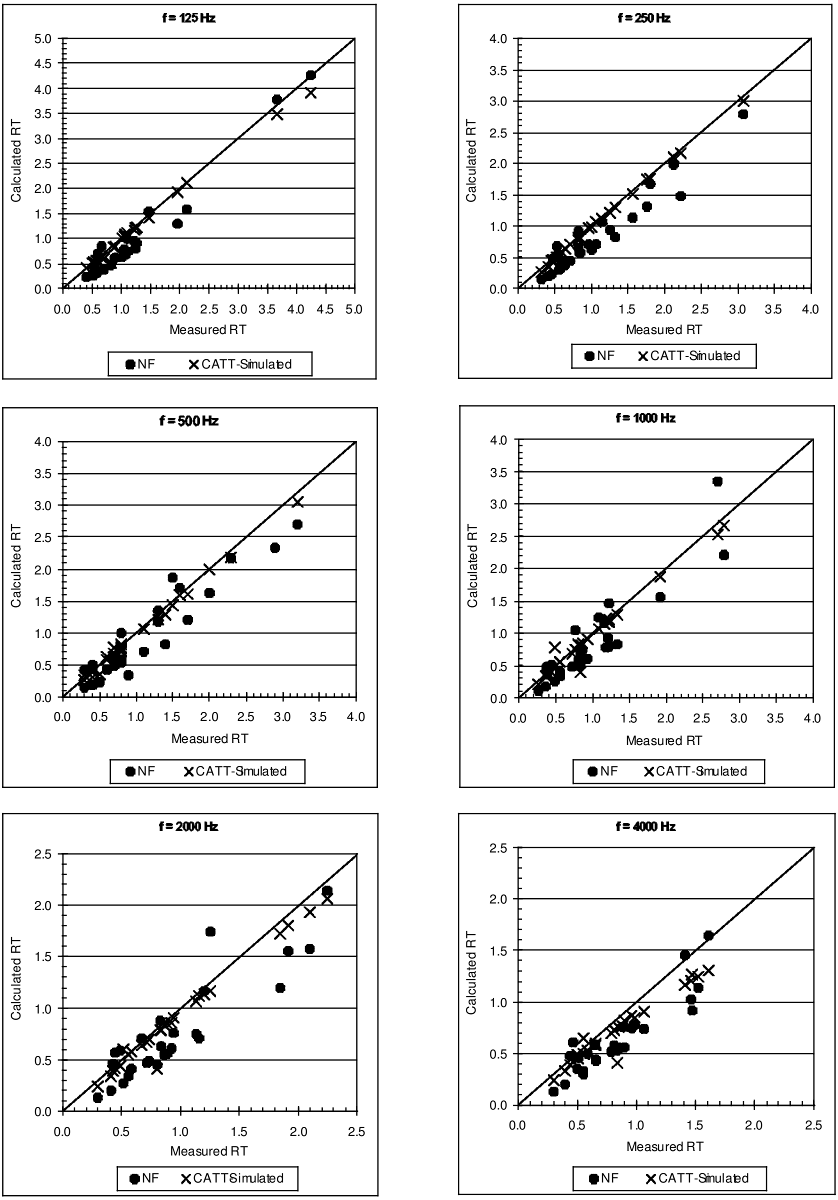

Figure 8. Calculated reverberation times using "calibrated" sound absorption coefficients. Shown are results using the Fitzroy-Kuttruff equation (NF) and computer simulated results, plotted against measured values.

Table 3. Mean and the standard deviation of the mean for the calculated RT using a computer simulation program. The calculation was executed six times with identical input data.

No	Volume	Reverberation Time T30 in s					
	m^3	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
1	52.13	1.20	1.25	1.28	1.14	1.23	0.78
2	62.17	4.23	2.07	1.48	1.12	0.84	0.61
3	68.03	2.17	1.79	2.02	1.90	1.79	1.20
4	73.03	0.57	0.53	0.38±0.01	0.41	0.45	0.44
5	94.24	0.84	0.78	0.74	0.66	0.64	0.57
6	114.96	1.18±0.02	1.42±0.05	1.48±0.06	1.18±0.02	1.11±0.02	0.83
7	131.06	0.55	0.51	0.45	0.46	0.50±0.01	0.44
8	138.90	0.66	0.47	0.45±0.01	0.39	0.46	0.44±0.01
9	149.13	1.88	2.14	1.61	1.18	0.85	0.68
10	156.92	0.59±0.01	0.85	0.76±0.02	0.75	0.71	0.57
11	161.08	0.55±0.01	0.59±0.01	0.72±0.01	0.85±0.01	0.91±0.01	0.78±0.01
12	194.04	1.21	0.89	0.65±0.01	0.61	0.63	0.55
13	196.91	1.01	0.98	1.06	0.91	0.84	0.75
14	203.00	0.41	0.73	0.75	0.82	0.90	1.01
15	220.11	1.46	1.75	2.17±0.01	2.53	1.98	1.27
16	241.87	0.51	0.66	0.78	0.85	0.86±0.01	0.80
17	270.00	1.12±0.02	1.31±0.03	1.29±0.03	1.28±0.03	1.10±0.01	0.82
18	323.97	0.56	0.38	0.35±0.01	0.37	0.40	0.40
19	323.97	0.54	0.36	0.35	0.21	0.31	0.29
20	334.96	1.22±0.01	1.17±0.01	0.95±0.01	1.27	1.70	1.23
21	379.01	0.98±0.01	0.99±0.01	0.85±0.02	0.72±0.01	0.71±0.01	0.62
22	750.00	1.09±0.01	0.99	1.14±0.01	0.88	0.74	0.61
23	750.00	1.22	1.24	0.88±0.01	1.17	1.04	0.91
24	1,100.55	3.57±0.02	2.95±0.01	2.95±0.01	2.54±0.01	1.83	1.11
25	1,899.68	0.64±0.01	0.54	0.57±0.01	0.81	0.75	0.74
26	1,899.68	0.74±0.01	0.50±0.01	0.57±0.01	0.66±0.01	0.64	0.63±0.01

In ray-based programs, the model requires a diffusion coefficient to be assigned to a given room surface. In general, however, it is a problem to obtain information about diffusion coefficients of surfaces[20] Therefore, the diffusion coefficient of individual surfaces was found by "calibrating" the diffuse model so that the results obtained approximately predicted the measured reverberation time. For this investigation it was found that diffusion coefficients of approximately 10 % up to 30 %, across the six octave-band centre frequencies from 125 Hz to 4 kHz, provided reasonable results compared to measured reverberation times. In most cases, a diffusion coefficient of 30% satisfied the diffuse conditions, to match measured reverberation times. It should be mentioned that no fine "tuning" was applied in matching the diffusion coefficient for obvious reasons. The same value for the diffusion coefficient was applied across all frequency bands, although the computer program used, had the capability to set frequency dependent diffusion coefficients. It turned out that the prediction result is highly sensitive to the value of diffusion coefficient used. Changing the diffusion coefficient from 10 % to 20 % and 30 % in some cases yielded significantly different reverberation times as can be seen in Figure 9. This is a serious problem in computer simulated room acoustic calculations and is the object of ongoing research.

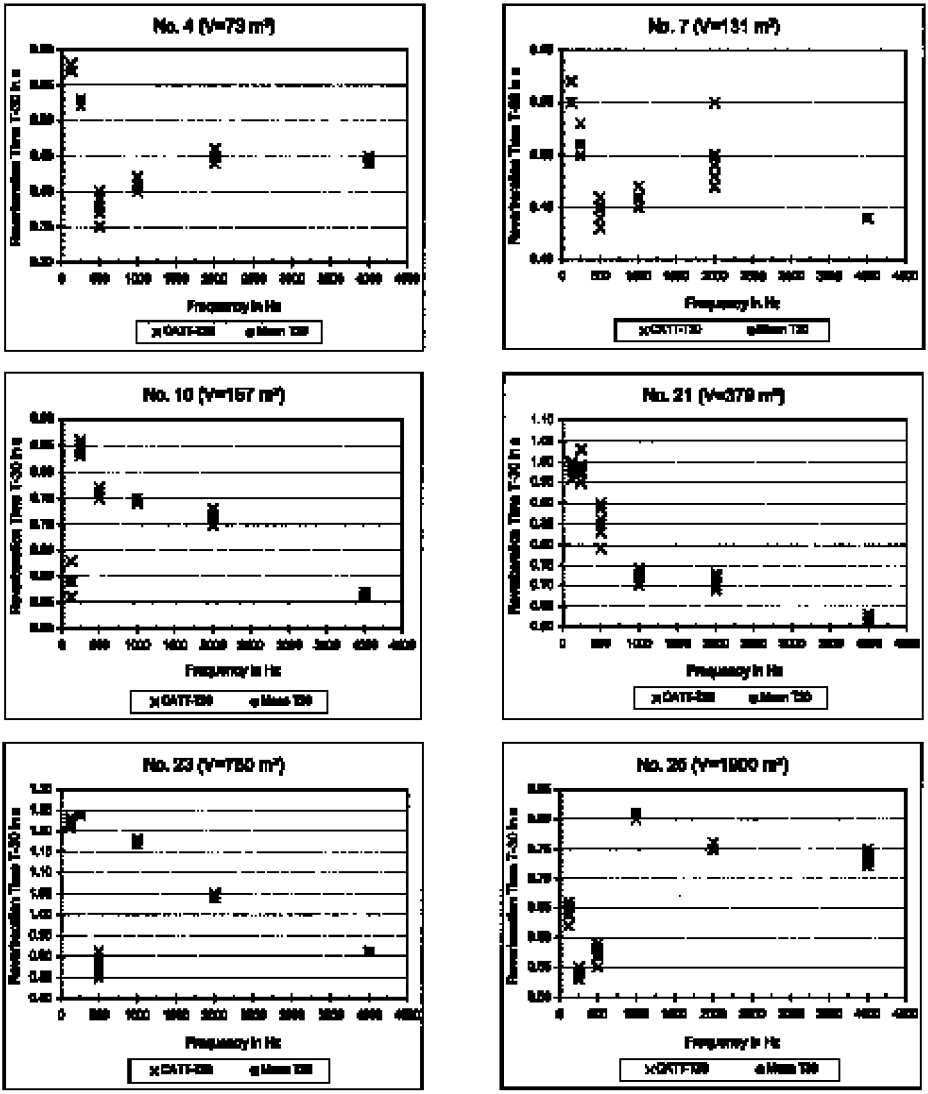

Figure 9. Examples of calculated RT for six rooms using a computer simulation program running six times for identical input data.

Another problem lies in the nature of ray tracing programs. The results obtained are, in general, not constant, i.e. different processing times for the calculation leads to different results. This is well known since diffusion is handled by a stochastic process. This is especially the case for non-diffuse absorption distribution where the statistical properties of the room reveals some differences in the calculation results. This may be seen in Figure 9 where some calculation results are presented showing the individual computed reverberation times for identical input data. All calculations for the 26 rooms were carried out six times and the mean and the standard deviation of the mean are calculated. The results are given in Table 3.

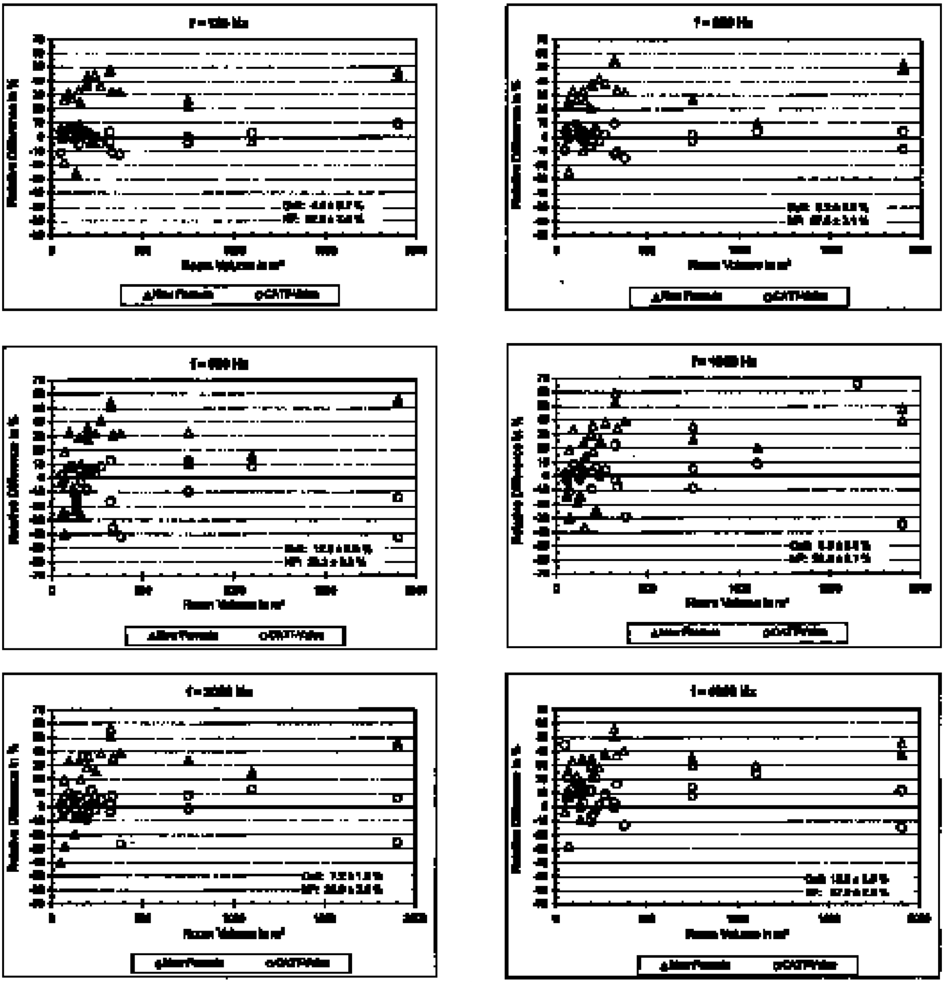

Figure 10. Relative difference of calculated reverberation times compared to the measured RT. Indicated in each graph is the mean for each room.

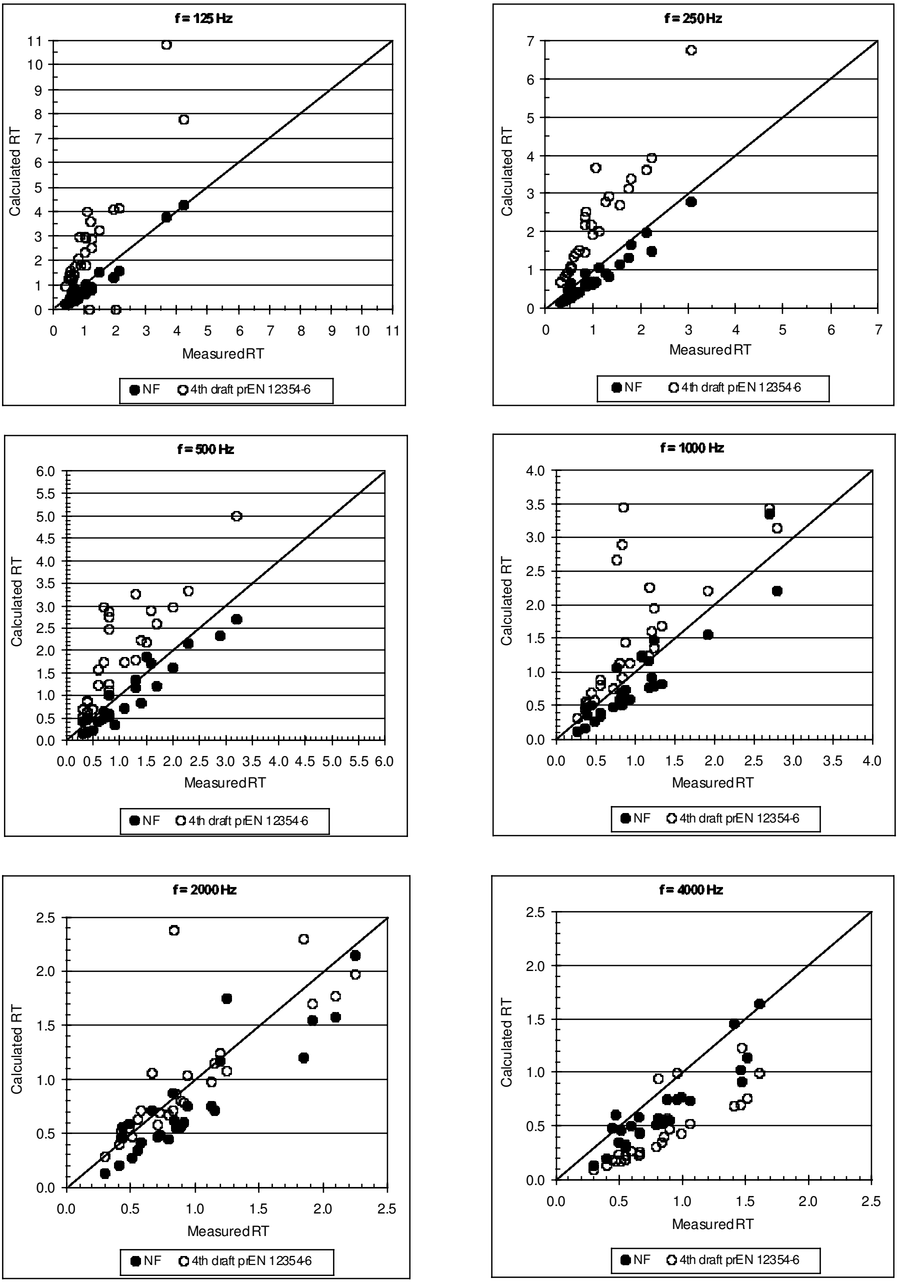

Figure 11. Comparison of calculated and measured reverberation time using the Fitzroy-Kuttruff equation and the proposed model of Annex D of prEN 12354-6 using “calibrated” sound absorption coefficients. The line shows measured reverberation time, plotted against itself.

For each room the calculation was done using identical input data. Figure 9 shows the mean of the results obtained for predicted reverberation times, for six rooms, across the six octave-bands from 125 Hz to 4 kHz. Figure 10 shows the differences in values calculated using the Fitzroy-Kuttruff equation and the values predicted by the CATT-Acoustic program.

It is apparent from Figure 10 that values predicted using the CATT-Acoustic program ranged from about 5 to 12%, and were on average 8%, above measured reverberation times, whereas the Fitzroy-Kuttruff equation gave approximately constant values across all frequency bands of 28%. The average difference between the CATT-Acoustic predicted value and the Fitzroy-Kuttruff value was calculated to be approximately 26%.

Recently the 4th draft of the European Standard prEN 12354-6[21] was published, which offers a method of calculating reverberation time in cases where the sound absorption in the room is non-uniformly distributed. Annex D indicates possible ways to improve predictions of enclosed spaces, which have non-regular, distributed sound absorption. Annex D of pr EN 1234-6 is based on a model proposed by Nilsson[22] to deal with rectangular rooms, by dividing the sound field into that part that is at grazing incidence to the surface considered, and that part that is non-grazing, taking due account of the different effect of absorbing materials for these different sound fields. The Annex to prEN 12354-6 provides a practical estimation, based on Nilsson's model, but making use of absorption data as measured according to standard methods. For the higher frequencies the total sound field is divided in three grazing fields, grazing to the surfaces perpendicular to each room axis and a diffuse field. For each of these fields the absorption and corresponding reverberation time are determined. For the lower frequencies the total sound field is considered. The absorption area for grazing modes A_x, A_y and A_z, the absorption area for low frequencies Axyad and the absorption area A_d for the diffuse field are given in Annex D of prEN 12354-6. Here, the equations are shown, neglecting the term for air absorption and the absorption area of objects:

$$A_x = \frac{0.5}{l_2}(A_{w1} + A_{w2}) \left(\frac{1000}{f}\right)^{1.5} + [A_{sidewall,1} + A_{sidewall,2} + A_{Floor} + A_{Ceiling}]\sqrt{\frac{2f}{1000}} \tag{16a}$$

$$A_y = \frac{0.5}{l^2}(A_{sidewall,1} + A_{sidewall,2}) \left(\frac{1000}{f}\right)^{1.5} + [A_{w1} + A_{w2} + A_{Floor} + A_{Ceiling}]\sqrt{\frac{2f}{1000}} \tag{16b}$$

$$A_z = \frac{0.5}{l^2}(A_{Floor} + A_{Ceiling}) \left(\frac{1000}{f}\right)^{1.5} + [A_{w1} + A_{w2} + A_{sidewall,1} + A_{sidewall,2}]\sqrt{\frac{2f}{1000}} \tag{16c}$$

$$A_{xyzd} = (A_{Floor} + A_{Ceiling} + A_{w1} + A_{w2} + A_{sidewall,1} + A_{sidewall,2})\sqrt{\frac{2f}{1000}} \quad (16d)$$

$$A_d = A_{w1} + A_{w2} + A_{sidewall,1} + A_{\mathrm{sidewall,2}} + A_{floor} + A_{\mathrm{ceiling}} \quad (16e)$$

The reverberation time for each sound field x, y, z and d, as given in Annex D is, by once again neglecting the object fraction term,

$$T_{x,y,z,d} = \frac{0,16V}{A_{x,y,z,d}} \quad (17a)$$

A global estimation of the effective reverberation time is given by the Annex D as

$$\mathrm{T_{eff} = (T_x + T_y + T_z + T_d) / 4} \quad (17b)$$

or for lower frequencies, and neglecting the object fraction term,

$$\mathrm{T_{eff} = 0,16\ V / A_{xyzd}} \quad (17c)$$

If the differences between the four reverberation times from Eq. 17a are small, the diffuse field reverberation time can be considered as an adequate estimation for the situation under consideration. If not, the effective reverberation time can be taken to be the more realistic estimate.[21]

In this present work, however, no detailed comparison is offered, but for comparison results are shown in Figure 11, using the calculation method of Annex D of the 4th draft of prEN 12354-6.

The improvement using the Fitzroy-Kuttruff equation method presented here, compared to the 4th draft of prEN 12354-6 is evident in Figure 11, across all frequency bands from 125 Hz to 4 kHz.

6. CONCLUSION

The present study has shown in detail a proposed modification of the Fitzroy equation. In the current evaluation of reverberation time prediction for non-uniformly distributed sound absorption in the rooms under investigation, it is found that the average difference of the Fitzroy-Kuttruff equation was typically less than 30 %. The investigation clearly shows that using the Fitzroy equation without any modification occasionally leads to unreasonable results. This is in agreement with results provided by Bistafa and Bradley.[11] The reason of this malfunction could not be identified, however, it turned out not to be a matter of absorption partitions in the room. A remarkable improvement was observed using the modified Kuttruff correction to the Fitzroy equation. The difference between Fitzroy and the Fitzroy-Kuttruff equation was observed in this investigation to lie approximately between 33 % - 42 %, with the Fitzroy-Kuttruff equation predicting values that are closer to measured reverberation times. In general, however, the Fitzroy-Kuttruff equation gives shorter reverberation times than those measured, although, under some conditions longer values result. The average difference was observed to be approximately 28%. Comparison of computer simulated results revealed an average difference of approximately 8%. This result is in good agreement with investigation results in the literature[11, 20] and lends support to the premise that the matched sound absorption coefficients and diffusion coefficients for the individual sound absorbent materials under investigation were appropriately chosen.

In order to contrast with "up-to-date" calculation methods, a comparison was made with reverberation times calculated using the method of the 4th draft of prEN 12354-6. The Fitzroy-Kuttruff equation revealed significantly better results.

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