PREDICTION OF REVERBERATION TIME IN RECTANGULAR ROOMS WITH NON UNIFORMLY DISTRIBUTED ABSORPTION USING A NEW FORMULA

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ABSTRACT

In this paper it will be presented a new formula, i.e. a modified Fitzroy-Kuttruff equation, for estimating the reverberation time in rooms with non uniformly distributed sound absorption. Calculated results are presented comparing different "up-to-date" formulae for the estimated reverberation time as well as for measured data. A modification of Fitzroy's equation is discussed in this paper and some practical examples are presented which compare predicted and measured values of reverberation time in real rooms.

INTRODUCTION

It is general known that Sabine's or Eyring's formula seriously in error if the sound absorption of the room is unevenly distributed. In 1988 Higini Arau [1] published a paper reporting a new formula for calculation of the reverberation time in rectangular rooms with non uniformly distributed sound absorption. Relating to the work of Fitzroy [2] it will be shown in this paper that the empirical Fitzroy equation can be modified applying Kuttruff's correction [3] originally related to Eyring's formula, yielding reverberation times which are close to measured values [4]. A suggested modification of Fitzroy's equation was first presented in [4] and was further discussed in [5]. Differences between results derived from Fitzroy's, Sabine's and Eyring's equation, as well as from Arau's formula and others, are compared to those obtained from measurements in real rooms.

A CORRECTION TO FITZROY'S FORMULA

Fitzroy's equation [2] assumes the Eyring concept [6] and considers the reverberation time of the room analogous to an area-weighted arithmetical mean of the reverberation time of the three room directions. On the other hand, Kuttruff [3, 7] introduced a correction to the Eyring formula and could show that his correction to the Eyring formula can easily be applied to the case where n-1 surfaces have nearly the same reflection coefficient and one surface, namely the nth surface, e.g. the floor where the audience sits over, a different reflection coefficient shows. His presented results showing a very good agreement with computer simulated results [7].

With the assumption that in real rooms the main absorption is always on the floor or at the ceiling or on both one can modify the Fitzroy equation into a more convenient equation for practical use. It is proposed to deal with the concept of a Kuttruff correction originally designed to correct the Eyring formula. Than it is possible to split the Kuttruff correction into two parts namely the part of ceiling-floor and the part of the walls.

THE MODIFIED FITZROY EQUATION

For the important practical case where either the ceiling and/or the floor is highly absorptive Fitzroy's equation may be written in a modified manner. Kuttruff presented in [3] the Eyring correction using the concept of the reflection coefficient:

$$\rho = 1 - \alpha$$

and presented an expression yielding an absorption exponent. Under the assumption that the absorption coefficient α and hence ρ are independent of the angles Kuttruff made use of Lambert's law of diffuse reflection. By focussing on the overall reverberation time, neglecting details of the decay process and under the assumption of an exponential law for the time dependence of the irradiation strength he defined an absorption exponent α^*, which is valid under the aforementioned assumption. The assumption of an exponential law is reasonable since, at least in rectangular rooms, the decay process of the sound energy will decrease exponentially [7, 8]. Kuttruff derived a correction to Eyring's formula and showed that the absorption exponent would assume its Eyring value if the irradiation strength were constant [7]

$$\alpha_{Eyring} = -\ln \overline{\rho} = -\ln (1-\overline{\alpha})$$

The absorption exponent is

$$\alpha^* = \ln\left(\frac{1}{\overline{\rho}}\right) + \ln\left(1 + \frac{\sum_n \rho_n \left(\rho_n - \overline{\rho}\right) S_n^2}{\left(\overline{\rho}\, S\right)^2 - \sum \rho_n^2 S_n^2}\right)$$

where $\overline{\rho} = 1 - \overline{\alpha}_n$, denotes the average reflection coefficient of surface area S_n

S is the total surface area of the room in m².

In most cases the second term in the denominator is much smaller than the first and hence can be neglected [7]. Expanding the second logarithm into a power series and neglect all terms of higher than first order gives

$$\alpha^* \approx \alpha_{Eyring} + \frac{\sum_n \rho_n \left(\rho_n - \overline{\rho}\right) S_n^2}{\left(\overline{\rho}\, S\right)^2}$$

Since the Eyring formula has to be modified it is reasoning modifying the Fitzroy equation in a similar manner. This modification leads to the modified Fitzroy equation (Fitzroy-Kuttruff equation).

We may rewrite Fitzroy's equation and introducing Kuttruff's modified correction yielding the New Formula

$$T_{60} = \left(\frac{0,32 * V}{S^2} \right) * \left(\frac{h(l+w)}{\overline{\alpha}^*_{ww}} + \frac{lw}{\overline{\alpha}^*_{CF}} \right)$$

where V, S = Volume in m³ and total surface area of the room in m²
h, l, w = room dimensions height, width and length in m
$\overline{\alpha}^*_{ww}$; $\overline{\alpha}^*_{CF}$ = average effective absorption exponent of walls, ceiling + floor

with
$$\overline{\alpha}^*_{ww} = -\ln(1-\overline{\alpha}) + \left[\frac{\rho_{ww}(\rho_{ww}-\overline{\rho}) * S^2_{ww}}{(\overline{\rho} * S)^2} \right]$$

$$\overline{\alpha}^*_{CF} = -\ln(1-\overline{\alpha}) + \left[\frac{\rho_{CF}(\rho_{CF}-\overline{\rho}) * S^2_{CF}}{(\overline{\rho} * S)^2} \right]$$

and $\overline{\alpha}$ is the arithmetic mean of the surface averaged absorption coefficient

$\rho = (1-\alpha)$ is the reflection coefficient

S_{CF} = 2lw; is the surface area of ceiling and floor, in m²

S_{WW} = 2lh + 2hw; is the surface area of the walls, in m²

S_{total} = 2[h(l + w) + lw)]; is the total surface area of ceiling, floor and walls, in m²

COMPARISON OF CALCULATED VALUES AND MEASURED RESULTS

A comparison of calculated results using different reverberation formulae under investigation are shown in the in Figure 1. The formulae under investigation in Figure 1 are Sabine, Eyring [6], Fitzroy [2], Millington-Sette [9, 10], Tohyama [11], Nilsson [12] and Arau [1].

NB: In this paper all presented figures are related to a mid frequency of 500 Hz and neglecting sound absorption of air.

It is interesting in some aspects to compare calculated reverberation time using different calculation formulae. In Figure 1 the calculated reverberation time is shown for different room volumes and overall surface absorptivity (averaged mean absorption coefficient) of 0,22-0,23. Comparison of predicted results with Sabine's reverberation time shows that Fitzroy, Tohyama, Arau and Nilsson always yield greater values. Whereas Eyring, Millington-Sette and Fitzroy-Kuttruff reveal smaller values than calculated by Sabine. The Nilsson model (prEN 12354-6) predicts, at least for the used date, unrealistic high reverberation time values, even longer values than predicted by Fitzroy. At this stage, it could not attained any explanation why Nilsson's model produces such high values.

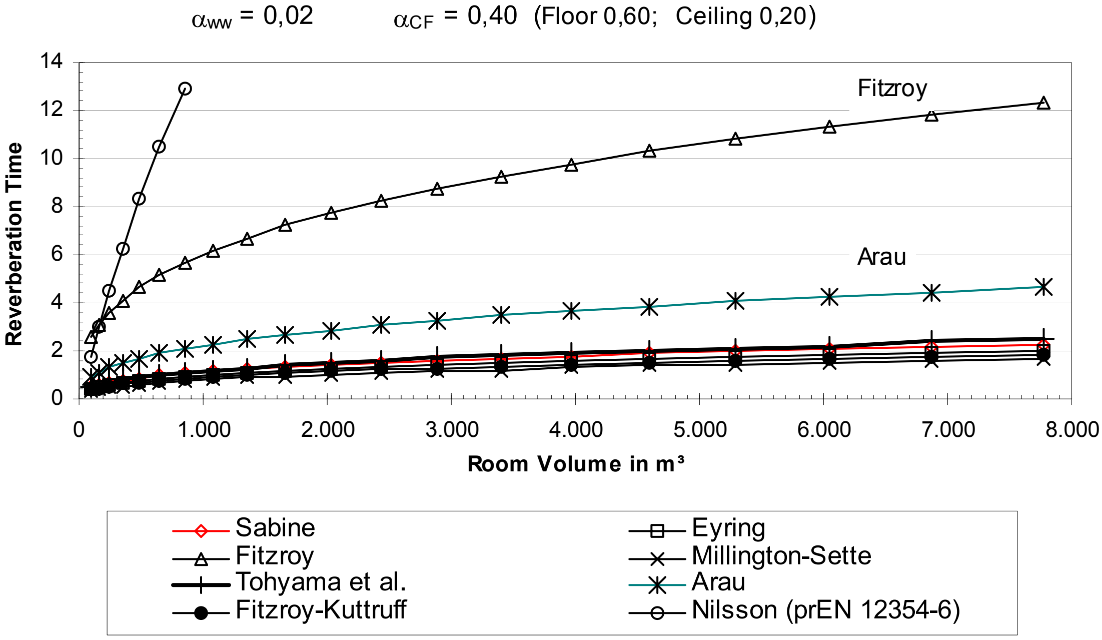

Figure 1. Calculated reverberation time for different reverberation time formulae.

In order to confirm the New Formula it is needed to compare with measured reverberation time values. Due to the fact that in practice, i.e. in real rooms, the sound absorption coefficient of the surfaces are not known it is assumed that from the measured reverberation time the appropriate values of the sound absorption of the individual surfaces can be matched using the theory of Sabine. This was done by "fitting" the calculated reverberation time using Sabine's formula and comparing the obtained results with the measured reverberation time. The individual sound absorption coefficients where then used calculating the respective reverberation time using different formulae under investigation. The "real" rooms where empty with some absorbent surfaces. No air absorption is taken into account for computing reverberation time. The results are presented in Figure 2.

It turned out for these data that the measured RT is always greater than the Eyring values. Fitzroy's equation yields about 75% greater and Millington-Sette's equation as well as the Fitzroy-Kuttruff equation yield about 96% smaller values compared to the measured values. Using Arau's equation yield about 93% and Tohyama's about 89% greater values than measured. Nilsson's model as given in prEN 12354-6 yields always greater values than measured and for the used data it turned out to be at least 95% greater than 50% and about 65% greater than 100% of the measured values. In the following table the differences using Arau and New Formula compared with measured date are presented.

N = 29	**Using Arau**	**Using Fitzroy-Kuttruff**
Results differ from measured RT ≤ ± 20%	58 %	62 %
RT < measured RT	27 %	96 %
RT > measured RT	62 %	3 %

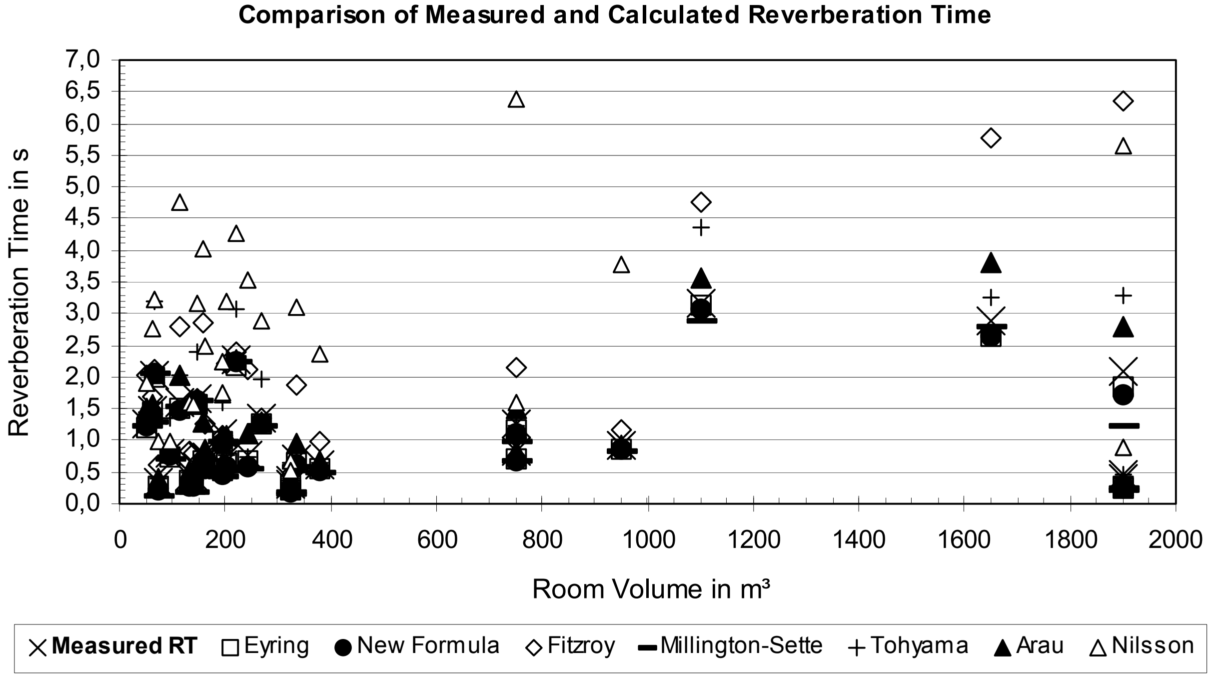

Figure 2. Calculated and measured reverberation time for different reverberation time formulae.

As a result of the foregoing data the difference between Arau's and Fitzroy-Kuttruff's value differ from each other in a range of ± 20% by about 44%. This means, the New Formula yields about 3% lower and about 93% greater values than using Arau's formula. For about 3% of the data there have been identical values obtained.

In Figure 3 it will be focused graphically on the comparison of measured results and predicted values using the formula of Arau and the New Formula.

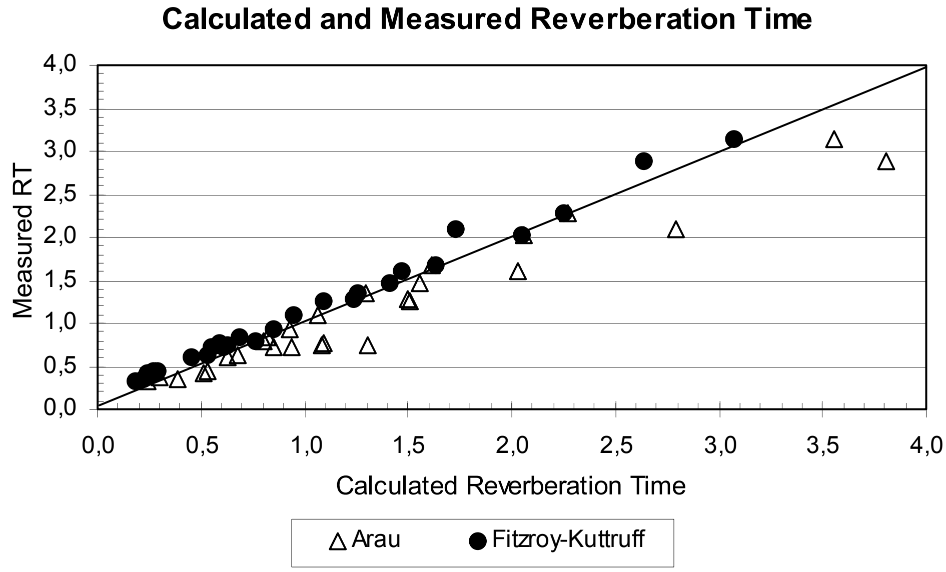

Figure 3. Comparison of calculated and measured reverberation times.

From Figure 3 it is seen that the measured reverberation time tends to higher values than the computed reverberation time using the New Formula (Fitzroy-Kuttruff) by use of "fitted" sound absorption coefficients, whereas Arau's value tends to provide greater values than measured. Arau's formula produced a greater difference for the measured value than it does using Fitzroy-Kuttruff's equation. The biggest difference of measured and predicted reverberation time using Arau's equation was a value of 44% greater and more than about 41% lower than measured. This range was observed similar but slightly smaller using the New Formula. In most cases, however, the difference using the Fitzroy-Kuttruff equation was less than about 20% of the measured value. A similar result is observed using the Eyring-Kuttruff equation. Since for both equations the "Kuttruff-correction" should be applied, it is worthwhile using the Fitzroy-Kuttruff equation.

SUMMERY

In this paper some of the most known formulae to predict the reverberation time are compared with calculated and measured values. It has been shown that the reverberation time according to a modified Fitzroy equation (Fitzroy-Kuttruff equation) is useful where the sound absorption at opposite sides are substantially higher than on the rest of the room surfaces. This is e.g. typically for offices where the assumption of diffuse field conditions for applying Sabine's theory are not in agreement with the existing absorber distribution. Comparison of calculated

Sabine and measured reverberation times revealed always too high values of computed reverberation times. In contrast, if one compares computed results using Eyring's formula and the Fitzroy-Kuttruff equation using "fitted" sound absorption coefficients, i.e. adequate absorption coefficients to match the measured reverberation time, revealed that Eyring' formula tends to predict, at least in this investigation, about 10 - 20% higher values than does using the Fitzroy-Kuttruff equation. In most practical cases where a "fitted" sound absorption coefficient on the basis of Sabine's theory is used to predict the reverberation time, in comparison with measured values, the difference between measurement and the New Formula (Fitzroy-Kuttruff equation) is less than 20%. Additionally, Arau's formula was compared which revealed that the New Formula provides values which are more than about 51% closer to the measured reverberation times.

BIBLIOGRAPHY

[1] Arau-Puchades, H.
"An Improved Reverberation Formula",
*Acustica, Vol. **65**, No. 4, 163-180 (1988)*

[2] Fitzroy, D.
"Reverberation formulae which seems to be more accurate with non-uniform distribution of absorption",
*The Journal of the Acoustical Society of America, Vol. **31**, 893-897 (1959)*

[3] Kuttruff, H.
"Nachhall und effektive Absorption in Räumen mit diffuser Wandreflexion",
*Acustica, Vol. **35**, No 3, 141-153 (1976)*

[4] Neubauer, R.O.
"Prediction of Reverberation Time in Rectangular Rooms with a Modified Fitzroy Equation",
ISSEM'99, 8th International Symposium on Sound Engineering and Mastering,
Gdansk, Poland, 115 - 122 (1999)

[5] Neubauer R.O.
Estimation of Reverberation Time in Rectangular Rooms with Non Uniformly Distributed Absorption Using a Modified Fitzroy Equation.
ICVS 7 Congress July 2000, Garmisch-Partenkirchen, Germany.

[6] Eyring, C.F.

"Reverberation Time in "Dead" Rooms",
The Journal of the Acoustical Society of America, Vol. ***1****, 217-241 (1930)*

[7] Kuttruff, H.
Room Acoustics,
3rd Edition 1991, Elsevier Science Publishers LTD

[8] Miles, R.N.
J. Sound Vibr., **92**, No. 2 (1984), 203

[9] Millington, G.
"A Modified Formula for Reverberation",
The Journal of the Acoustical Society of America. Vol. **4**, (1932), pp 69-82

[10] Sette, W.H.
"A New Reverberation Time Formula",
The Journal of the Acoustical Society of America. Vol. **4**, (1933), pp 193-210

[11] Tohyama, M., Suzuki, H., and Ando Y.
The Nature and Technology of Acoustic Space,
Academic Press, London, (1995)

[12] European Standard prEN 12354-6. February 2000, Annex D. (Nilsson's Model)
Building Acoustics. Estimation of acoustic performance of buildings from the performance of elements,
Part 6: Sound absorption in enclosed spaces.CEN/TC126/WG2 – N216