SEVENTH INTERNATIONAL CONGRESS
ON SOUND AND VIBRATION
4-7 July 2000, Garmisch-Partenkirchen, Germany

ESTIMATION OF REVERBERATION TIME IN RECTANGULAR ROOMS WITH NON UNIFORMLY DISTRIBUTED ABSORPTION USING A MODIFIED FITZROY EQUATION

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Abstract

In many practical cases the assumption of diffuse field conditions for applying Sabine's theory are not in agreement with the existing absorption distribution. The sound field will be, in general, sufficiently diffuse if there are not large differences in the basic dimensions of the room, the walls are not parallel, the sound absorbing material is uniformly distributed, and most internal surfaces are divided into parts. In practice, almost all of these requirements are not fulfilled. Therefore Sabine's formula as well as other classical reverberation equations like Eyring's or Millington-Sette's formula cannot be accurately applied. In 1959 Dariel Fitzroy published a paper devoted to the problem of a more accurate calculation of the reverberation time with non uniformly distributed absorption. Fitzroy stated that the presented new formula afforded results which are closer to those measured in real halls. In real rooms Fitzroy's equation may therefore become a useful design tool for estimating reverberation time, however, only if it is modified. A modification of Fitzroy's equation is discussed in this paper and some practical examples are presented which compare predicted and measured values of reverberation time in real rooms.

INTRODUCTION

In 1959 Dariel Fitzroy published a paper devoted to the problem of a more accurate calculation of the reverberation time with non uniformly distributed absorption. In his paper Fitzroy stated that the presented new formula afforded results which are closer to those measured in real halls or if predicted on the basis of the numbers of times the energy is reflected during a 60-dB decay, reducing the energy by the absorptive coefficient at each reflection, using the mean free path as the average path, i.e. with the Eyring formula. Although the presented results were quite notable no further recognition was applied in the last 40 years. For practical use in designing a room with a desired reverberation time one have to ensure an appropriate combination of room volume and interior finishes. The reverberation time is predicted for the planned volume and surface materials of the auditorium. In starting the prediction of reverberation time, one should bear in mind that the Sabine equation, as well as other classical reverberation equations, is valid only when the sound field in the room can be considered to be sufficiently diffuse. The most common cause of reduced diffusion is extensive sound absorbing finishes of the ceiling and the floor, and having walls that are all vertical, poorly subdivided, and that reflect sound well. In such rooms, sound waves propagating in directions close to horizontal attenuate slowly, forming a horizontal reverberant field. The reverberation time appears to exceed the value calculated using Sabine's equation. Since this is a common situation in real rooms, Fitzroy's equation may become a useful design tool. A suggested modification of Fitzroy's equation was first presented in [1] and is further discussed in this paper. Differences between results derived from Fitzroy's, Sabine's and Eyring's equation are compared to those obtained from measurements in real rooms.

FITZROY'S EQUATION

Fitzroy states in [2] when one considers the possibilities suggested by physical acoustics, as contrasted with the geometrical concept, it is in general, the sound field may tend to settle into a pattern of simultaneous oscillation along a rectangular room's three major axes - the vertical, the transverse, and the longitudinal. The solution appeared to lie in some relationship within the three possible basic decay rates along these axes, each being influenced by the differing average absorptivities normal to these axes, each rate being unique with its specific axis. In a rectangular room, we are presented with three sets of parallel boundaries. If the energy oscillates simultaneously between each pair of boundaries the average absorption in each pair would control the sound wave excursions between that specific pair during the sound decay period. Since the total area of the room's boundaries encounters the energy field and influences the decay time to a degree depending upon absorptivity and location, each pair of parallel boundaries represents a specific percentage of the total. The difference between the Eyring formula and the Fitzroy equation is, that Eyring's formula assumes the absorptivity to be equal in all direction and the Fitzroy's equation, in the case of the rectangular room, three different calculations are made by means of the Eyring formula. But the average absorption is changed for each pair of boundaries. Therefore Fitzroy established ratios relating each pair of boundary areas to the total room areas. He proposed using only these percentages of the reverberation periods for the respective pairs of boundaries. These are added together and the resulting equation yields the reverberation time. Fitzroy's empirically derived equation which considers non-uniform distribution of absorption is

$$T_{60} = 0,16 * \frac{V}{S^2}\left[\frac{-x}{\ln(1-\alpha_x)} + \frac{-y}{\ln(1-\alpha_y)} + \frac{-z}{\ln(1-\alpha_z)}\right] \qquad (1)$$

where x, y, z are the total areas of two opposite parallel walls in m²

α_x, α_y, α_z are the average absorption coefficients of a pair of opposite walls

S is the total surface area of the room in m².

EYRING-KUTTRUFF-EQUATION

C.F. Eyring pointed out in his paper [3] that Sabine's formula is essentially a "live" room formula and that the reverberation time equation varies somewhat with the shape of the room. Eyring presented the revised theory thoroughly and derived a form of reverberation time equation which is more general than Sabine's formula. He presented the difference between the basic assumptions leading to the Sabine's and his formula as well as some experimental data which support the more general type of his equation. Eyring indicated also in his paper that no one formula, i.e. the Sabine's and Eyring's formula, are essentially all inclusive which means they need some modification. Eyring's formula which is extensively discussed in the literature is

$$T_{60} = \frac{0,16 * V}{-S \ln(1-\overline{\alpha})} \tag{2}$$

with $\overline{\alpha} = \frac{1}{S} \sum_i S_i \alpha_i$ (3)

Eyring's equation has been derived on the assumption that the sound field in the enclosure considered is ideally diffuse. Usually, it yields sufficiently correct results, although it is based upon the assumption, that equally sized wall areas are hit by the same energy amount per second. This is the case if the sound field in the room is diffuse or homogeneous which means that in each pint the directional distribution of energy transport is uniform. Since this condition would result in vanishing net energy flow, perfect sound field diffusion is incompatible with any wall absorption, strictly speaking, which causes at least some energy flow the sound source towards the wall. On the other hand, a non-uniform distribution of wall absorption may impair sound field homogeneity so severely that no correct results from application of Eyring's formula can be expected.

Kuttruff presented in [3] under the assumption of a diffuse sound reflection from each wall element a more general description of sound propagation in a room. He uses the concept of the reflection coefficient

$$\rho = 1 - \alpha \tag{4}$$

Under the assumption that the absorption coefficient α and hence ρ are independent of the angles he made use of Lambert's law of diffuse reflection. By focussing on the overall reverberation time, neglecting details of the decay process and under the assumption of an exponential law for the time dependence of the irradiation strength he defined an absorption exponent α^*. Kuttruff derived a correction to Eyring's formula and showed that the absorption exponent would assume its Eyring value

$$\alpha_{Eyring} = - \ln \overline{\rho} = -\ln(1-\overline{\alpha}) \tag{5}$$

if the irradiation strength were constant. This is true if ρ and hence the absorption coefficient α has everywhere the same value. In general, however, the effective absorption exponent will be smaller or larger than ($-\ln \overline{\rho}$), depending on the room shape and on the distribution of the wall absorption. Kuttruff's assumption using Lambert's law of diffuse reflections and Gilberts iteration scheme lead to the mentioned correction formula which is shown in detail in [4].

$$\alpha^* = \ln\left(\frac{1}{\bar{\rho}}\right) + \ln\left(1 + \frac{\sum_n \rho_n \left(\rho_n - \bar{\rho}\right) S_n^2}{\left(\bar{\rho}\, S\right)^2 - \Sigma \rho_n^2 S_n^2}\right) \tag{6}$$

where $\bar{\rho} = 1 - \bar{\alpha}_n$, denotes the average reflection coefficient of surface area S_n

S is the total surface area of the room in m².

In most cases the second term in the denominator is much smaller than the first and hence can be neglected. Expanding the second logarithm into a power series and neglect all terms of higher than first order yields

$$\alpha^* \approx \alpha_{Eyring} + \frac{\sum_n \rho_n \left(\rho_n - \bar{\rho}\right) S_n^2}{\left(\bar{\rho}\, S\right)^2} \tag{7}$$

Inserting Kuttruff's correction to the Eyring's reverberation formula and complete this formula by taking into account the attenuation constant m of air leads to

$$T_{60} = \frac{0,161 * V}{S\left[-\ln(1-\bar{\alpha})\right] + \Delta + 4\,mV} \tag{8}$$

where Δ is

$$\Delta = \frac{\sum_i \rho_i \left(\rho_i - \bar{\rho}\right) S_i^2}{\left(\bar{\rho}\, S\right)^2 - \sum_i \rho_i^2 S_i^2} \tag{9}$$

Kuttruff could show that his correction to the Eyring formula can easily be applied to the case where i-1 surfaces have nearly the same reflection coefficient and one surface, namely the ith surface, e.g. the floor where the audience sits over, a different reflection coefficient shows. His presented results showing a very good agreement with computer simulated results.

A CORRECTION TO FITZROY'S FORMULA

Since Kuttruff [4] introduced a correction to the Eyring formula and Fitzroy's equation [2] assumes the Eyring concept, it is proposed to deal with the same correction. With the assumption that in real rooms the main absorption is always on the floor or at the ceiling or on both one can modify the Fitzroy equation into a more convenient equation for practical use without loosing significant accuracy. Than it is possible to split the Kuttruff correction into two parts namely the part of ceiling-floor and the part of the walls. In the following paragraph the results are presented using a modified Kuttruff correction leading to a modified Fitzroy equation.

THE MODIFIED FITZROY EQUATION

Because the examples Fitzroy gives in the paper work so well with the three term formula, it is worthwhile attempting it. Compared calculation results reveal, however, no advantage over the more simple equation provided by Sabine or Eyring. Particular, calculating differently rooms, i.e. others than Fitzroy provides in his paper, results reveal the other way around. Using the Fitzroy equation provides much more higher values of reverberation time than using Sabine or Eyring. For the most important practical case where either the ceiling and/or the floor is highly absorptive we have an "almost two-dimensional" field. This occur in general in real rooms when the absorbing capacity of the ceiling and floor exceeds that of the walls, Fitzroy's equation may than be written in a modified manner. If we divide the floor and ceiling area and the rest of the wall areas we get expressions as follows.

$$S_{CF} = 2lw \tag{10 a}$$
$$S_{WW} = 2lh + 2hw \tag{10 b}$$
$$S_{total} = 2[h(l + w) + lw)] \tag{10 c}$$

Using above Eq. 10 a to 10 c we may rewrite Fitzroy's equation and introducing Kuttruff's modified correction of Eq. 12 yielding the New Formula

$$T_{60} = \left(\frac{0,32 * V}{S^2} \right) * \left(\frac{h(l+w)}{\overline{\alpha}^*_{ww}} + \frac{lw}{\overline{\alpha}^*_{CF}} \right) \tag{11}$$

where V, S = Volume in m³ and total surface area of the room in m²
h, l, w = room dimensions height, width and length in m
$\overline{\alpha}^*_{ww}$; $\overline{\alpha}^*_{CF}$ = average effective absorption exponent of walls, ceiling + floor

with
$$\overline{\alpha}^*_{ww} = -\ln(1-\overline{\alpha}) + \left[\frac{\rho_{ww}(\rho_{ww} - \overline{\rho}) * S^2_{ww}}{(\overline{\rho} * S)^2} \right] \tag{12 a}$$

$$\overline{\alpha}^*_{CF} = -\ln(1-\overline{\alpha}) + \left[\frac{\rho_{CF}(\rho_{CF} - \overline{\rho}) * S^2_{CF}}{(\overline{\rho} * S)^2} \right] \tag{12 b}$$

and $\overline{\alpha}$ is the arithmetic mean of the surface averaged absorption coefficient
$\rho = (1-\alpha)$ is the reflection coefficient

A comparison of calculated results using the New Formula and Eyring's formula are shown in the next graph. It should be mentioned that for all figures shown in this paper are for mid frequencies of 500 Hz and neglecting sound absorption of the air. In the Figure 1 the calculated reverberation time is shown for different room volumes and overall surface absorptivity of 0,02 (live room condition). Eyring's reverberation time is about 1% less than the reverberation time using the New Formula (Fitzroy-Kuttruff-Equation). On the other hand, Eyring yields about 1% less than using Sabine's formula. This states, that Sabine and the New Formula reveals the same value of reverberation time if the overall absorption of the room is little.

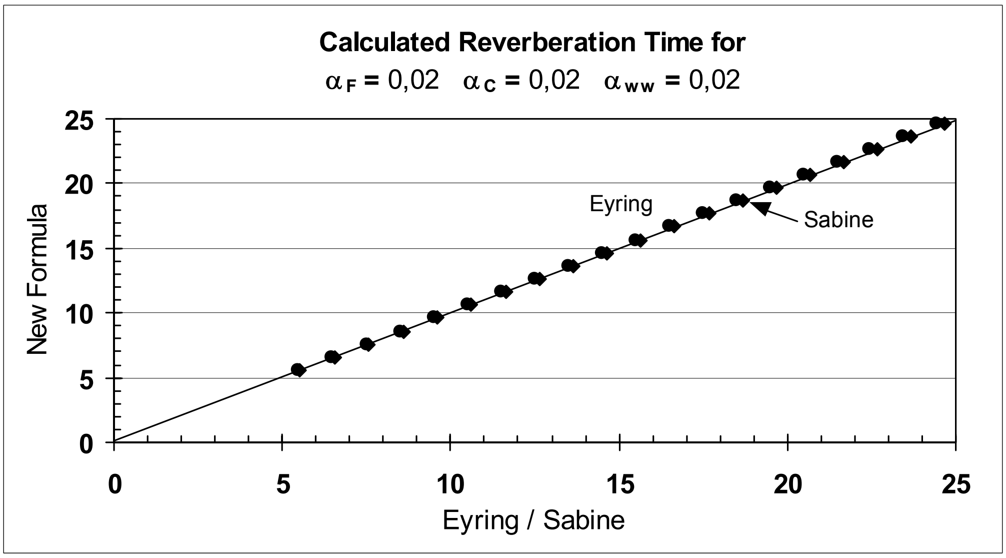

Figure 1. Calculated reverberation time for little absorption of all surfaces.

A comparison of results for "dead room condition", which means a overall surface absorptivity of 0,95, are shown in Figure 2. Eyring's reverberation time is close to the New Formula, whereas Sabine yields about 68 % higher values than using Eyring's formula.

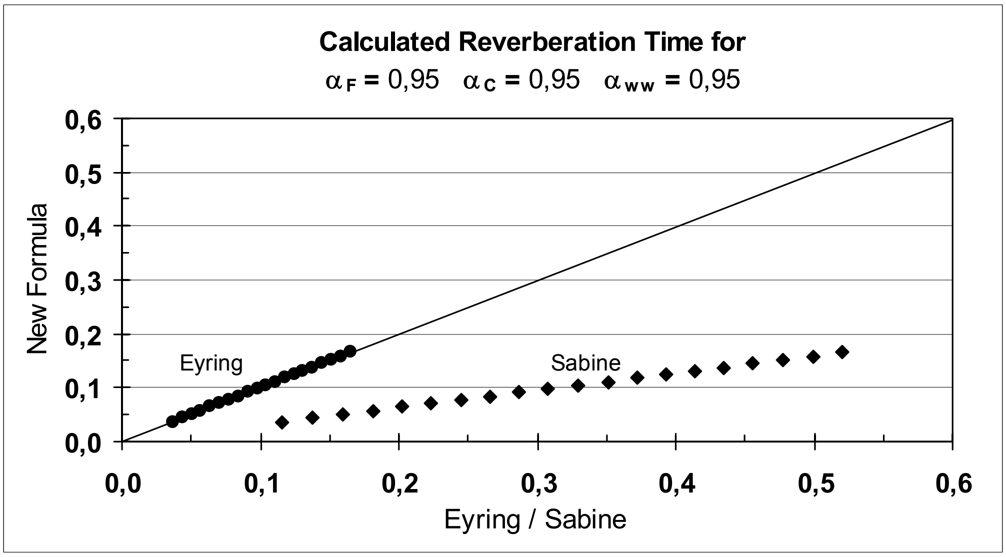

Figure 2. Calculated reverberation time for high absorption of all surfaces.

The effect of non-uniformly distributed sound absorption in rooms is shown in Figure 3.

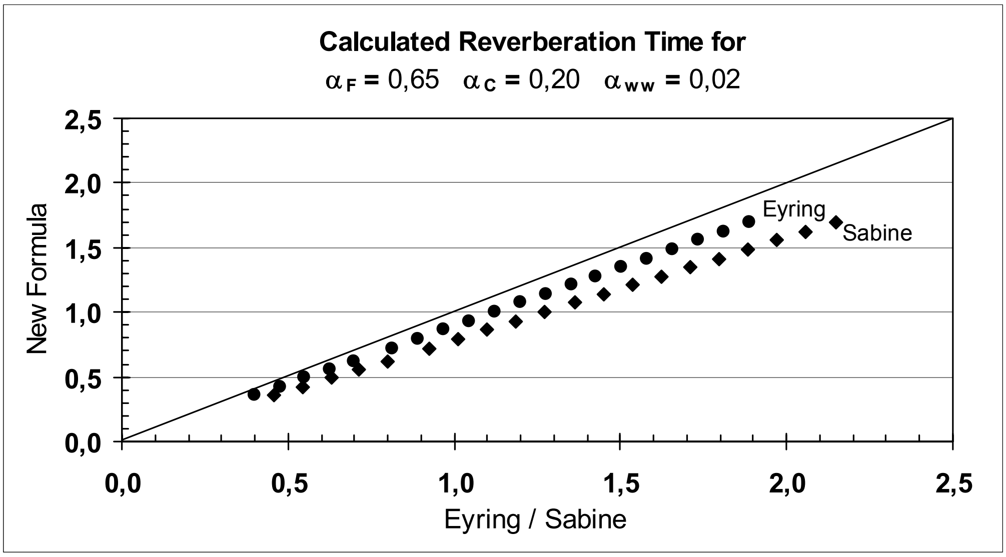

Figure 3. Calculated reverberation time for non-uniformly distributed sound absorption.

It is informative in some respect comparing measured reverberation time values and computed results using the New Formula (Fitzroy-Kuttruff-Equation). This comparison is shown in the next graph. In order to compare measured values in "real" rooms with computed values it is necessary to estimate the individual sound absorption coefficients of respective surfaces of the room. This was done by "fitting" the calculated reverberation time using Sabine's formula and comparing the obtained results with the measured reverberation time. The individual sound absorption coefficients where then used calculating the respective reverberation time using different formulae under investigation. The "real" rooms where empty with some absorbent surfaces. No air absorption is taken into account for computing reverberation time.

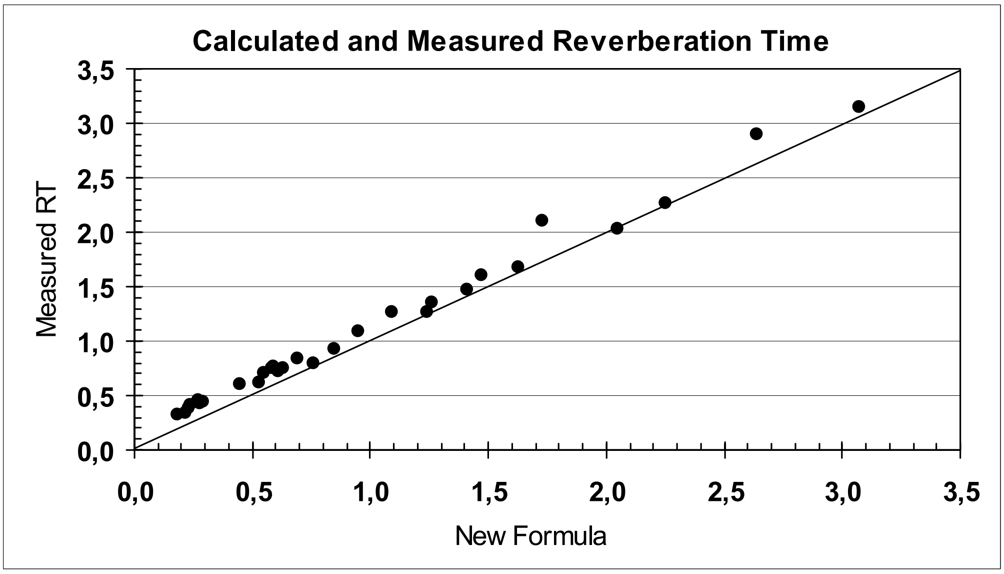

Figure 4. Measured and calculated reverberation time with "fitted" absorption coefficient.

From Figure 4 it is seen that the measured reverberation time tends to higher values than the computed reverberation time using the New Formula with “fitted” sound absorption coefficients. In most cases the difference is less than about 20%. A similar result is observed using the Eyring-Kuttruff equation. Since in both equations the “Kuttruff-correction” has to be applied, it is worthwhile using the Fitzroy-Kuttruff equation. This is because the Fitzroy-Kuttruff equation enables one to investigate the effect of applying different surfaces with different sound absorptivity.

SUMMERY

It has been shown that the reverberation time according to a modified Fitzroy equation is useful where the sound absorption at opposite sides are substantially higher than on the rest of the room surfaces. This is e.g. typically for offices where the assumption of diffuse field conditions for applying Sabine’s theory are not in agreement with the existing absorber distribution. In most practical cases where a “fitted” sound absorption coefficient on the basis of Sabine’s theory is used to predict the reverberation time, in comparison with measured values, the difference between measurement and New Formula is less than 20%. The investigations has clearly shown, that using the Fitzroy equation without any modifications leads to unreasonable results. Comparison of calculated Sabine’s and measured results always revealed too high computed reverberation times. In contrast, if one compares computed results using Eyring’s formula and the Fitzroy-Kuttruff equation using “fitted” sound absorption coefficients, i.e. adequate absorption coefficients to match measured reverberation time, reveals that Eyring yields in most cases of this investigation about 10 - 20% higher values.

LITERATURE

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